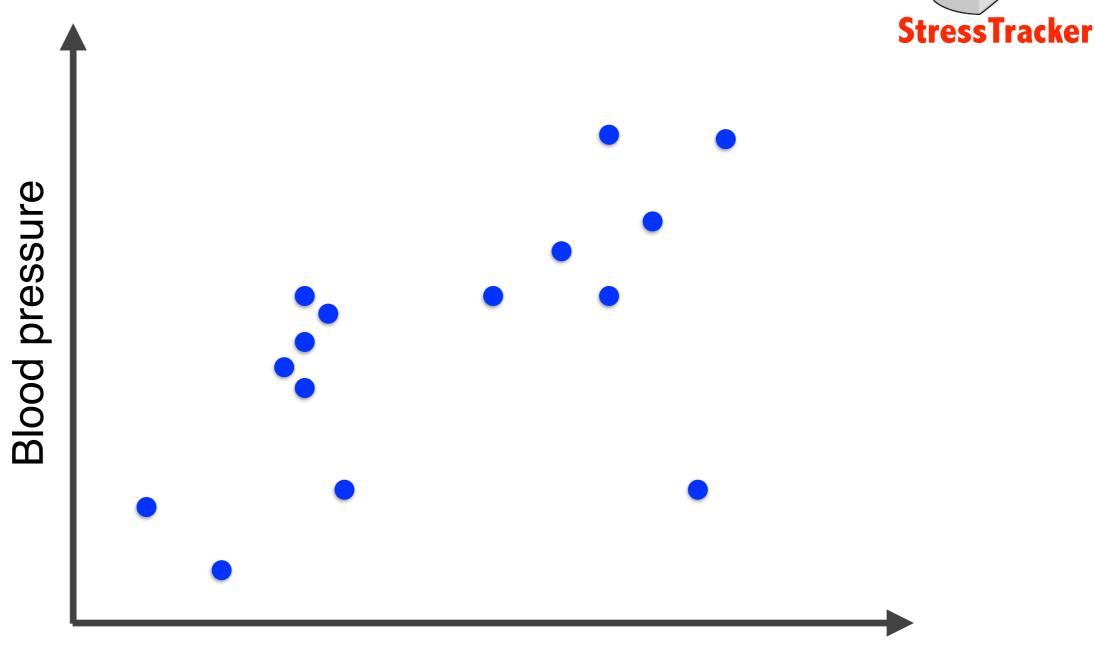
# Prio: Private, Robust, and Efficient Computation of Aggregate Statistics

#### Henry Corrigan-Gibbs and Dan Boneh Stanford University

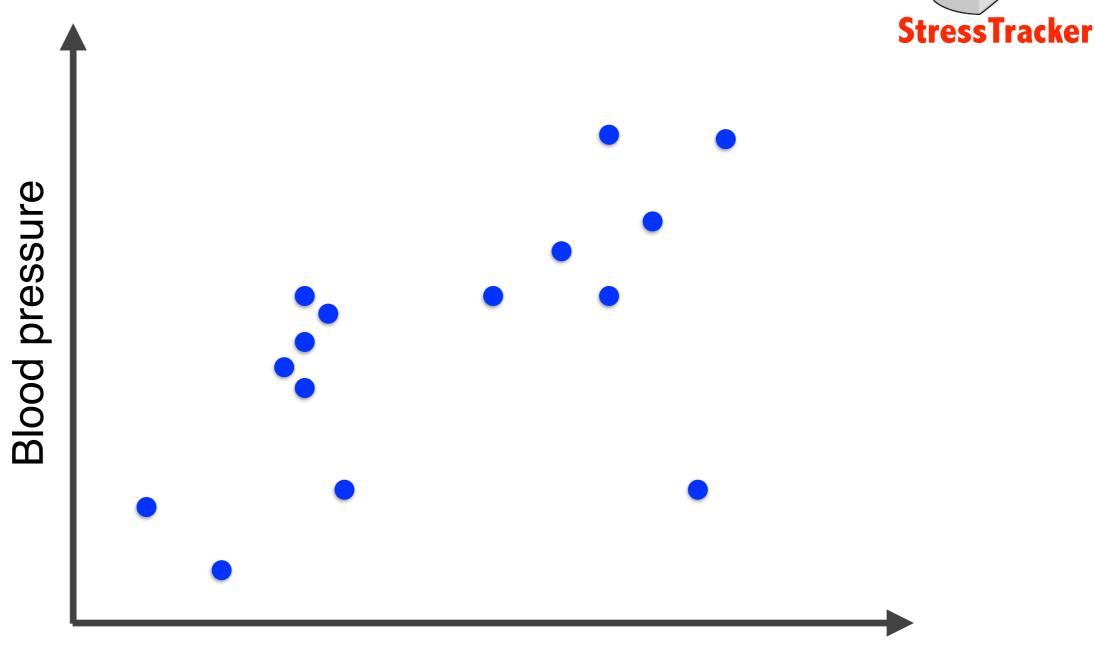
NSDI 2017

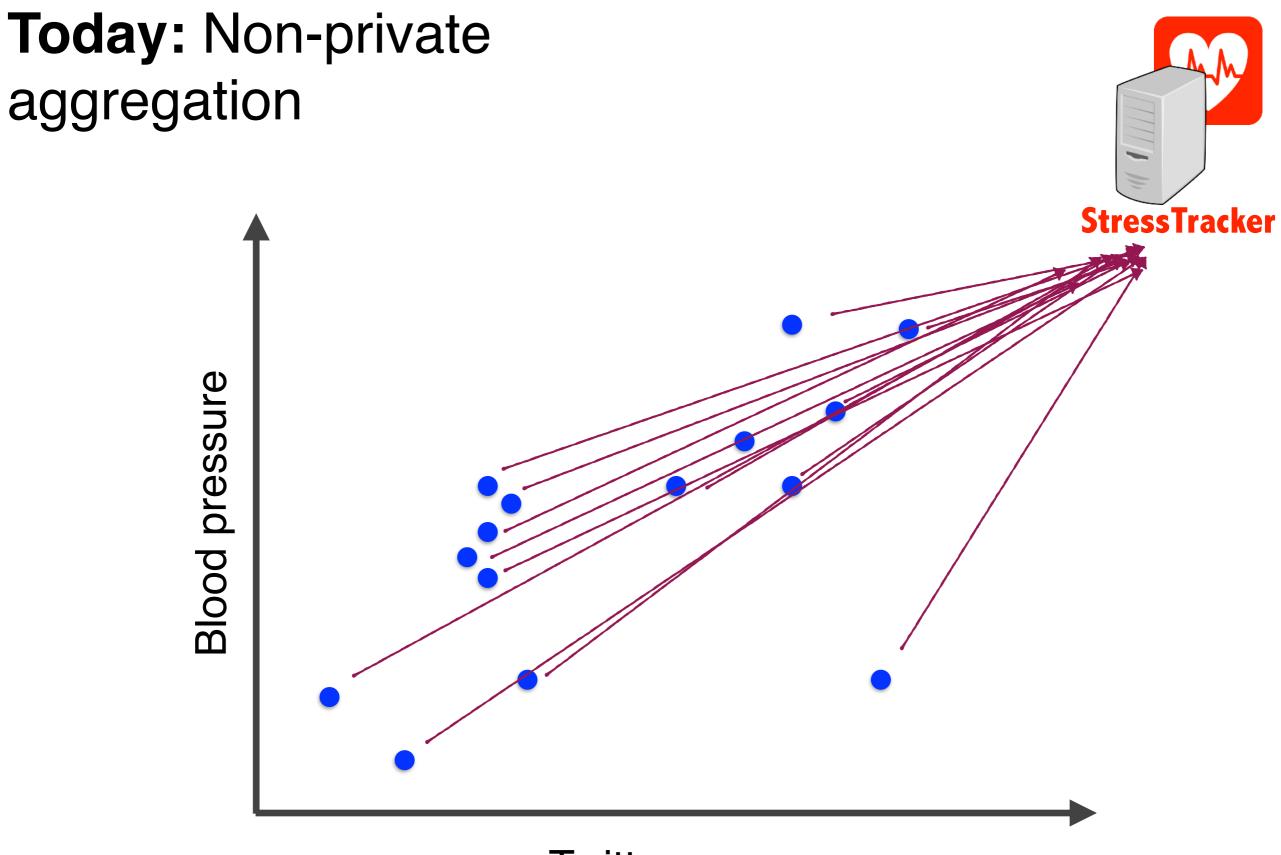
# **Today:** Non-private aggregation

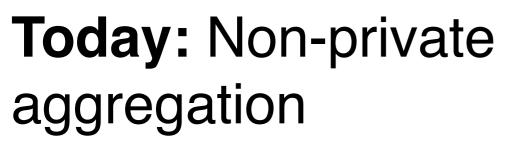


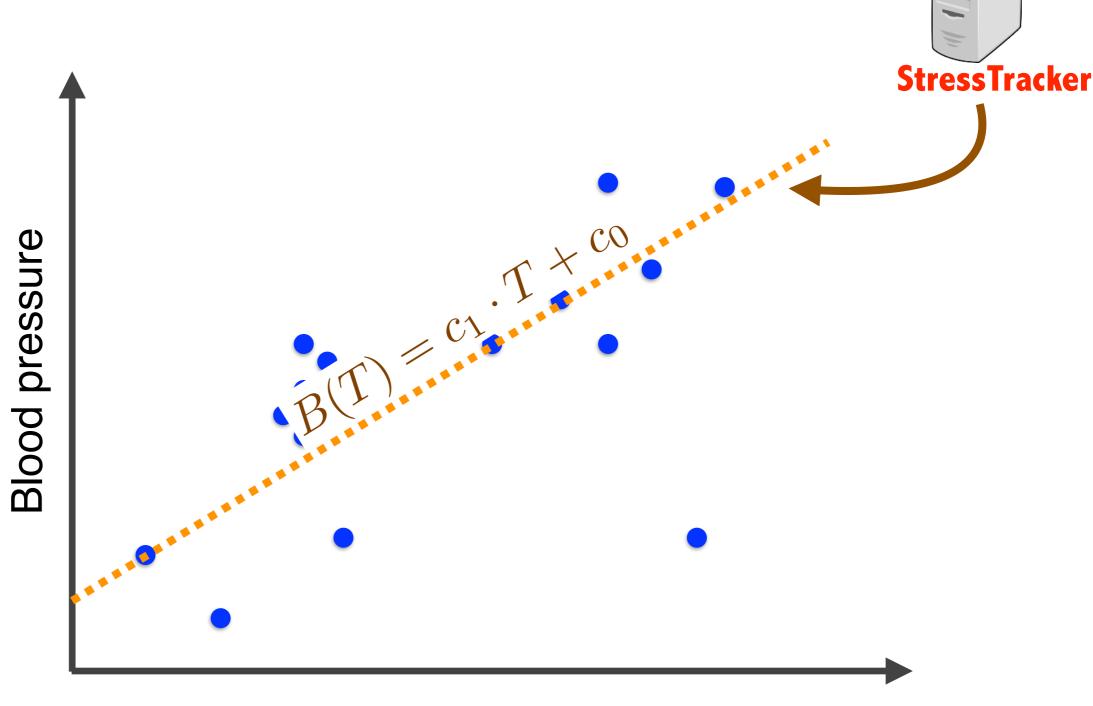
# **Today:** Non-private aggregation **StressTracker** Blood pressure Each user has a private data point

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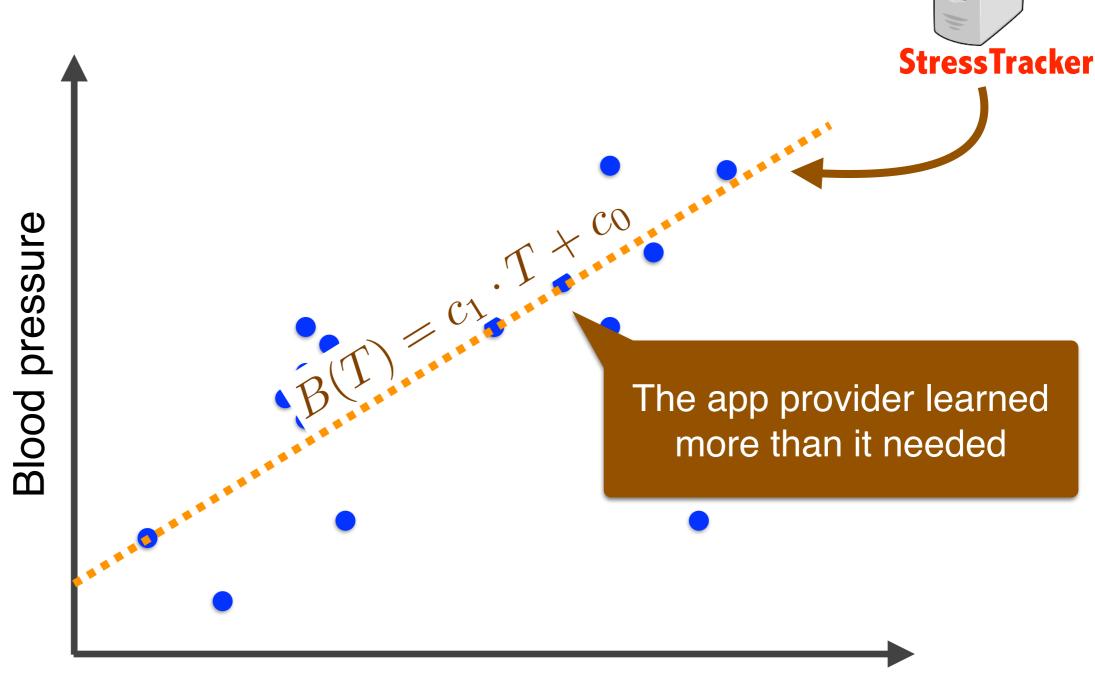




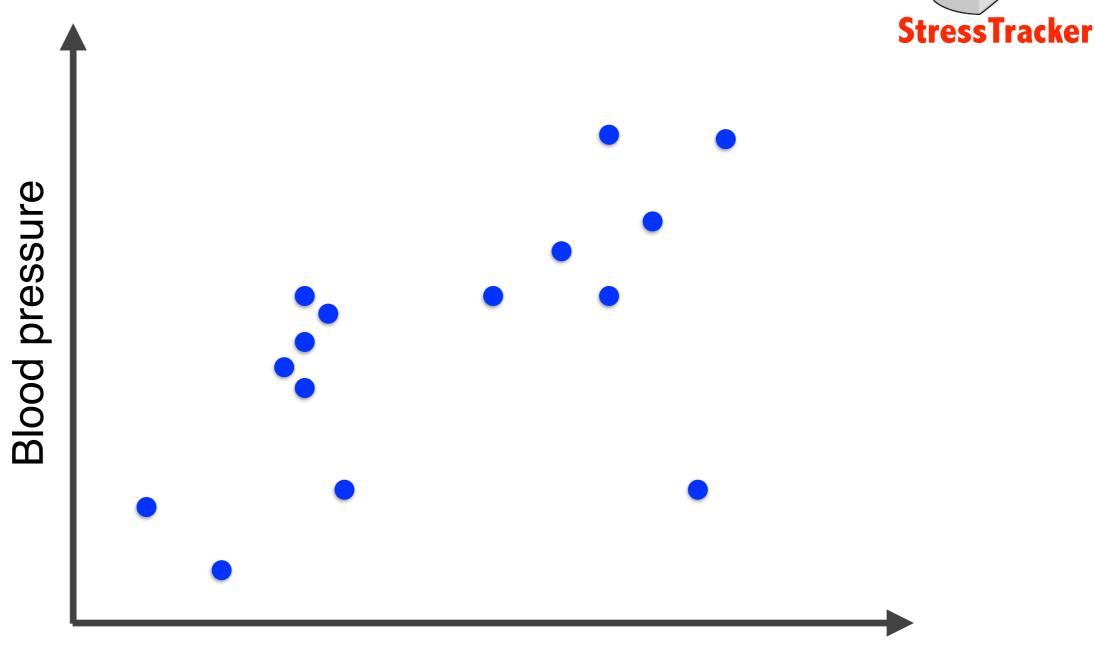


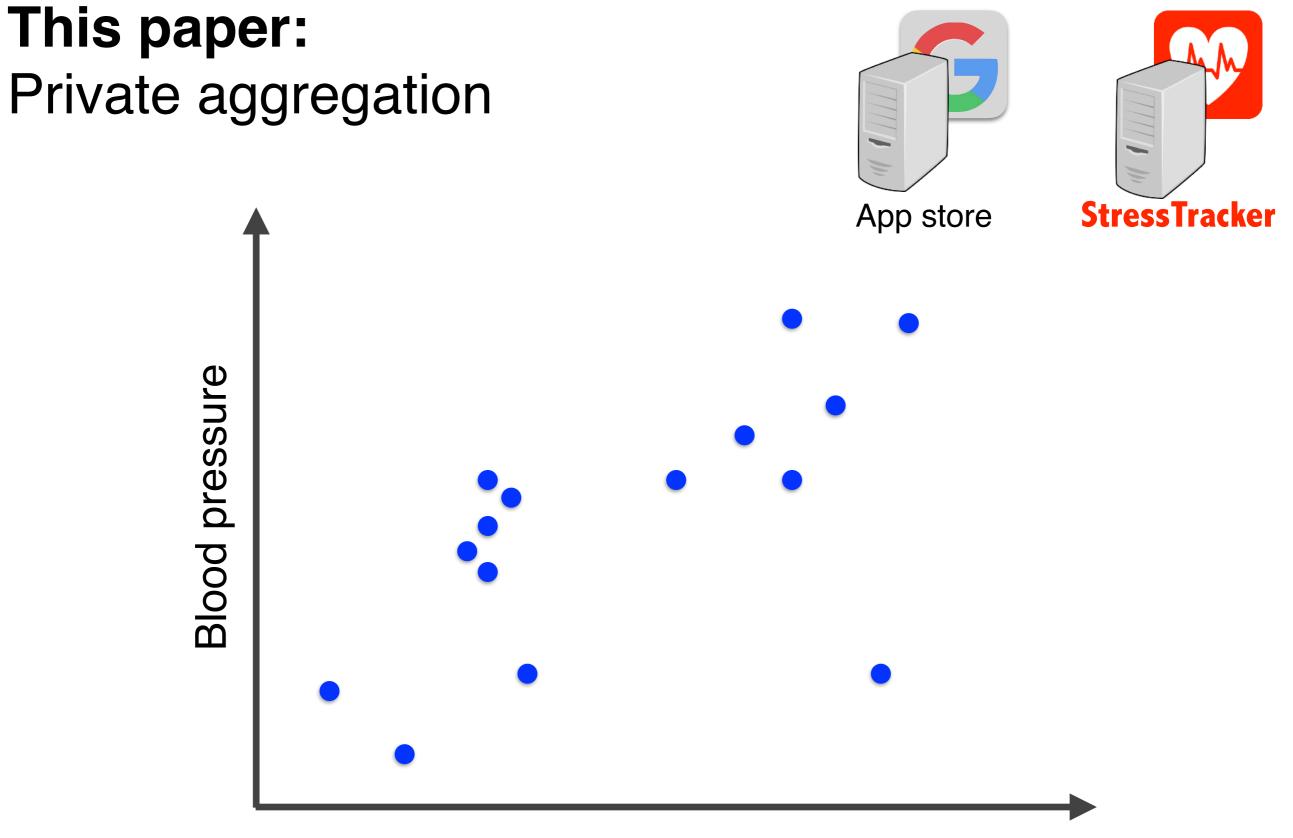


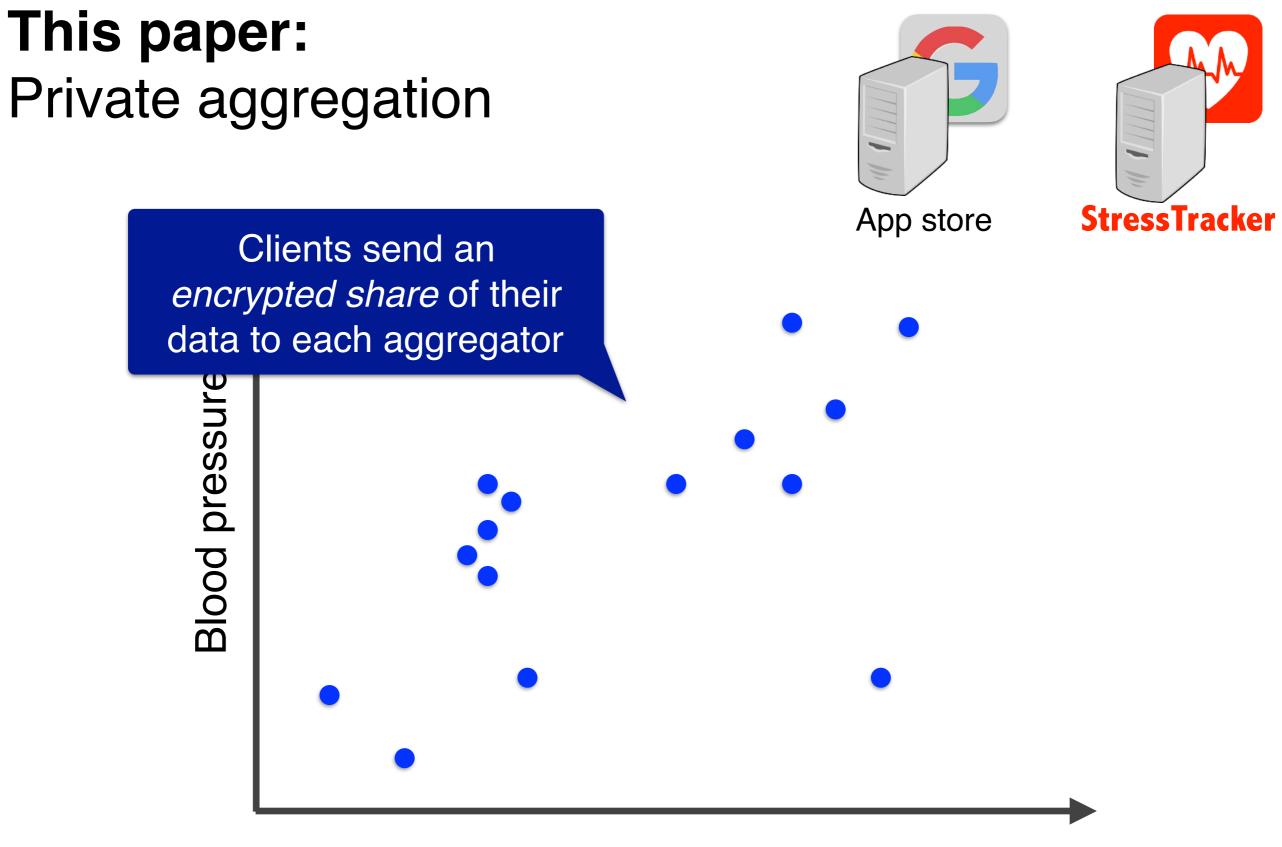
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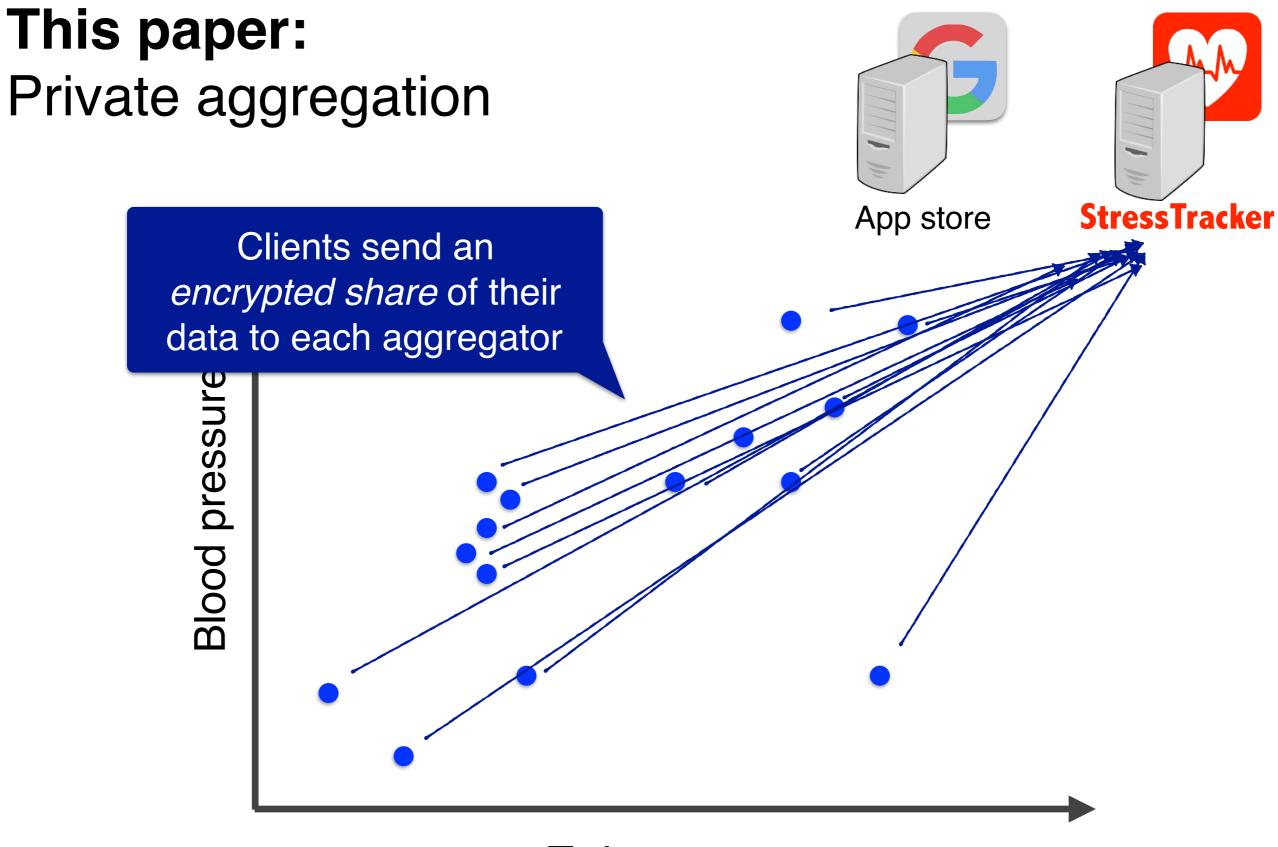


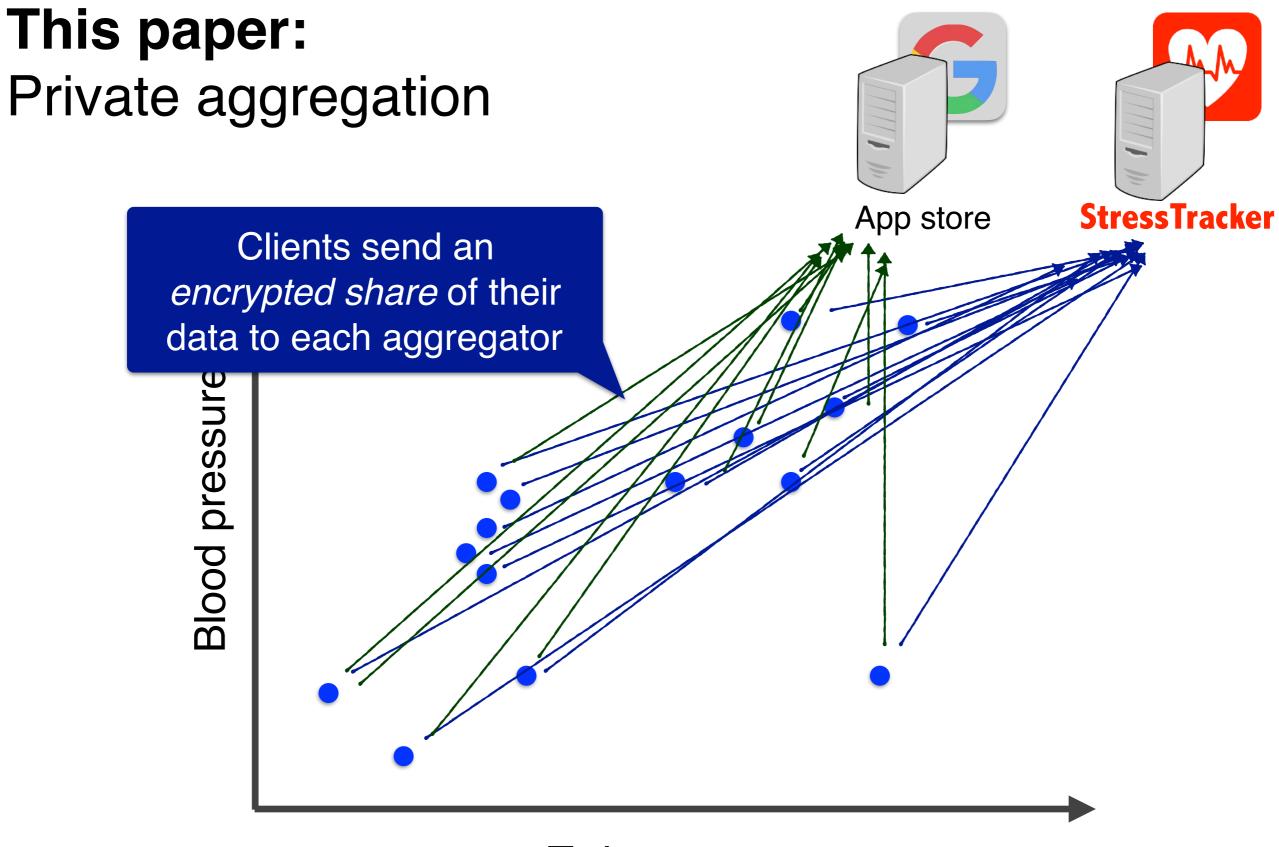
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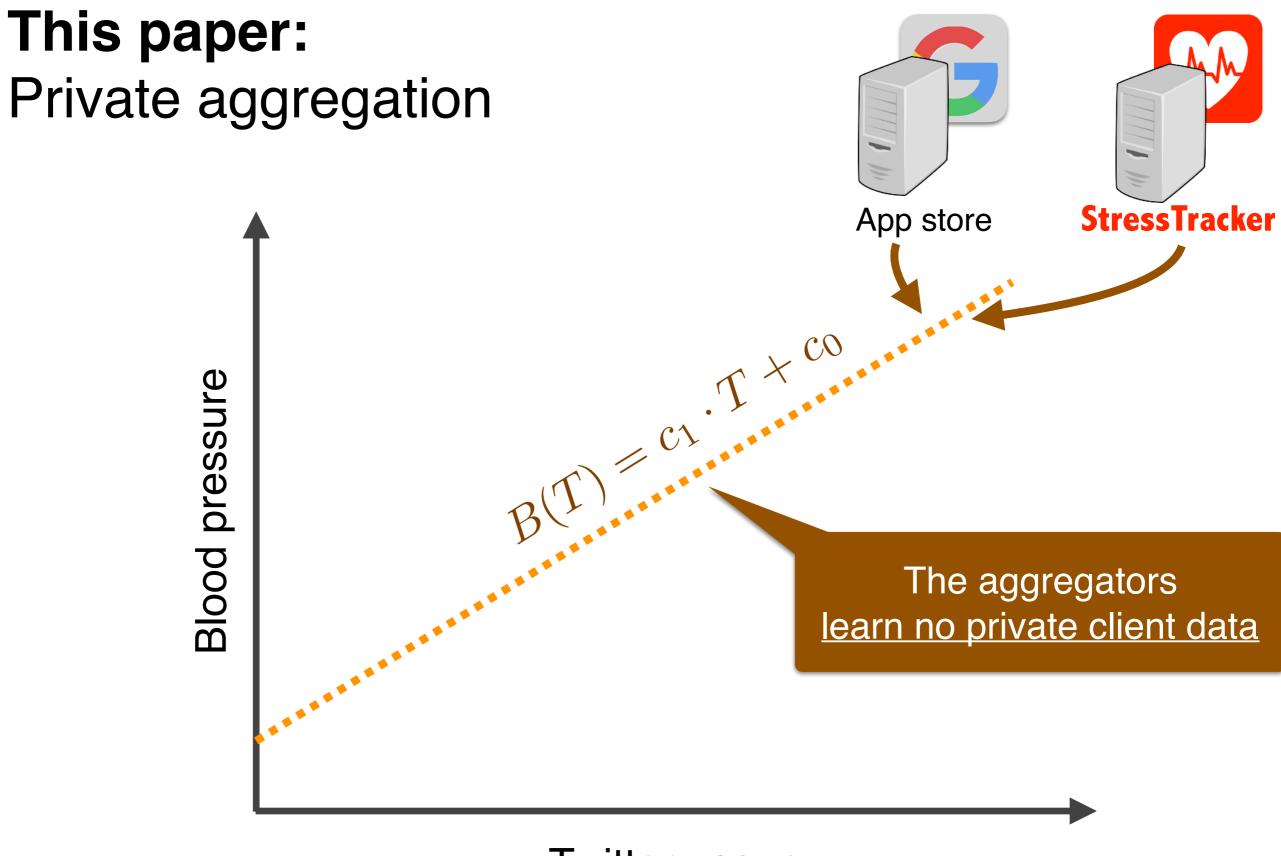






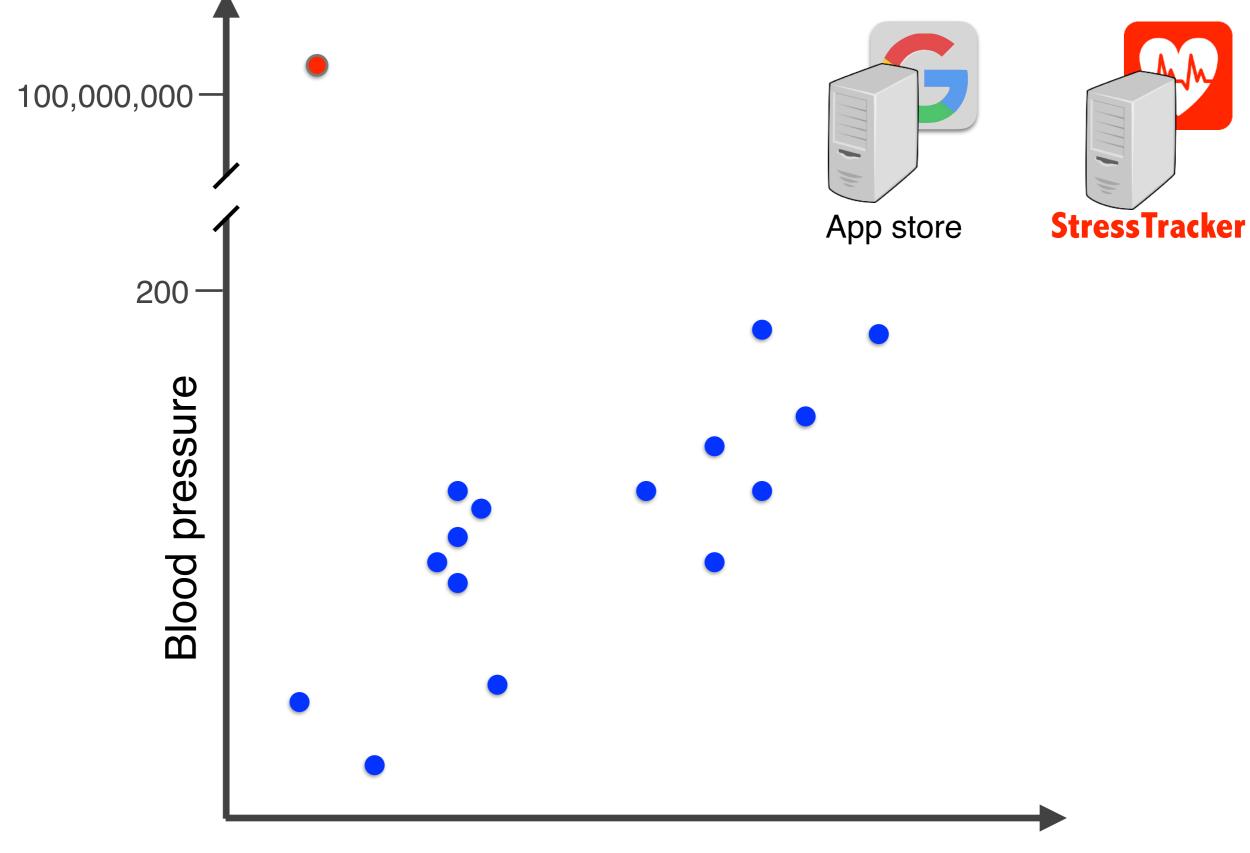


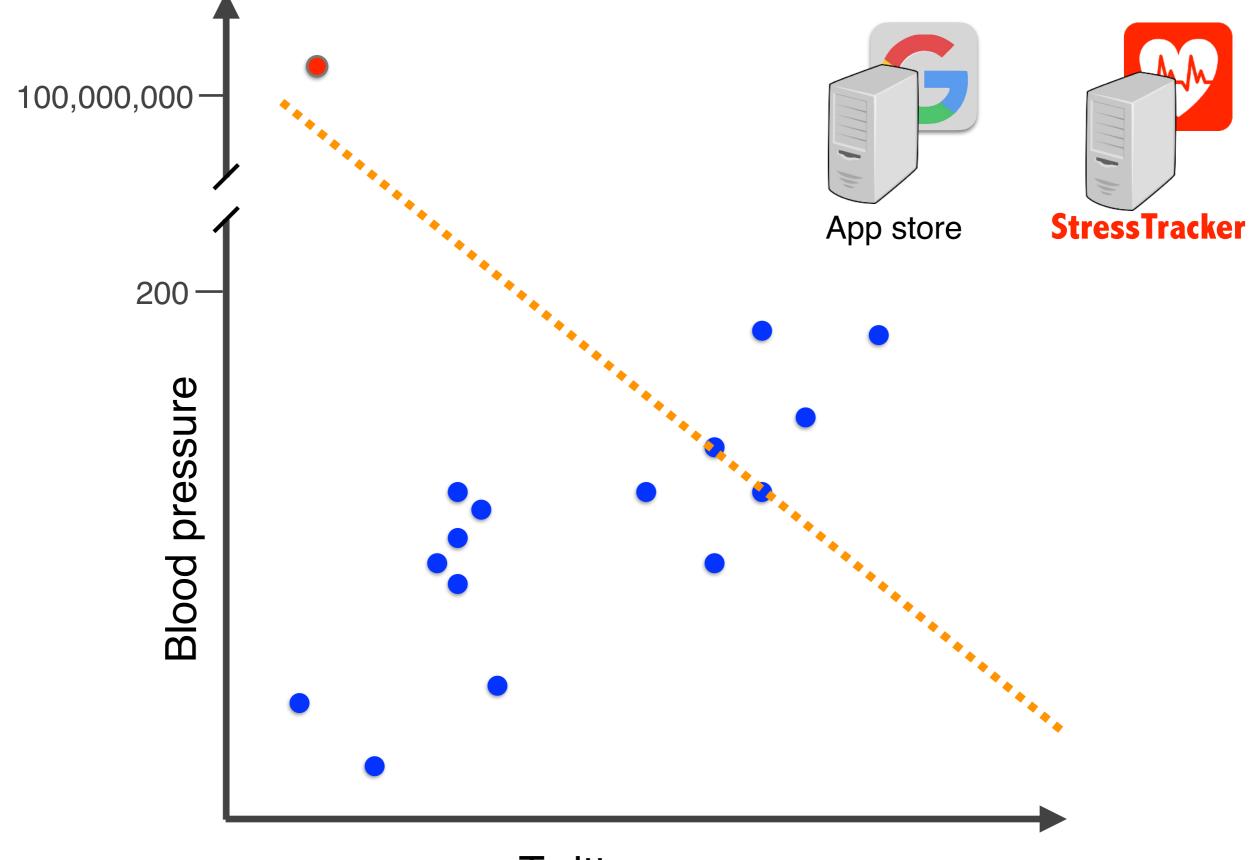






- **1. Exact correctness** If <u>all servers</u> are honest, servers learn  $f(\cdot)$
- **2. Privacy** If <u>one server</u> is honest, servers learn only\*  $f(\cdot)$
- **3. Robustness** Malicious clients have bounded influence
- 4. EfficiencyNo public-key crypto (apart from TLS)1000s of submissions per second







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No public-key crypto 1000s of submissior ...and Prio supports a wide range of aggregation functions f( • )

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## Contributions

### 1. Secret-shared non-interactive proofs (SNIPs)

- Client proves that its encoded submission is well-formed
- We do not need the power of traditional "heavy" crypto tools

#### 2. Aggregatable encodings

Can compute sums privately  $\implies$  Can compute f(·) privately ...for many f's of interest

# Related systems

- Additively homomorphic encryption
   P4P (2010), Private stream aggregation (2011), Grid aggregation (2011),
   PDDP (2012), SplitX (2013), PrivEx (2014), PrivCount (2016),
   Succinct sketches (2016), ...
- Multi-party computation [GMW87], [BGW88]
   FairPlay (2004), Brickell-Shmatikov (2006), FairplayMP (2008), SEPIA (2010), Private matrix factorization (2013), JustGarble (2013), ...
- Anonymous credentials/tokens VPriv (2009), PrivStats (2011), ANONIZE (2014), ...
- Randomized response [W65], [DMNS06], [D06] RAPPOR (2014, 2016)

#### **Prio is the first system to achieve**

exact correctness, privacy, robustness, efficiency.

# Outline

- Background: The private aggregation problem
- A straw-man solution for private sums
- Providing robustness with SNIPs
- Evaluation
- Encodings for <u>complex</u> aggregates

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- Every device i holds a value  $x_i$
- We want to compute

 $f(\mathbf{X}_1, \ldots, \mathbf{X}_N) = \mathbf{X}_1 + \ldots + \mathbf{X}_N$ 

without learning any users' private value x<sub>i</sub>.

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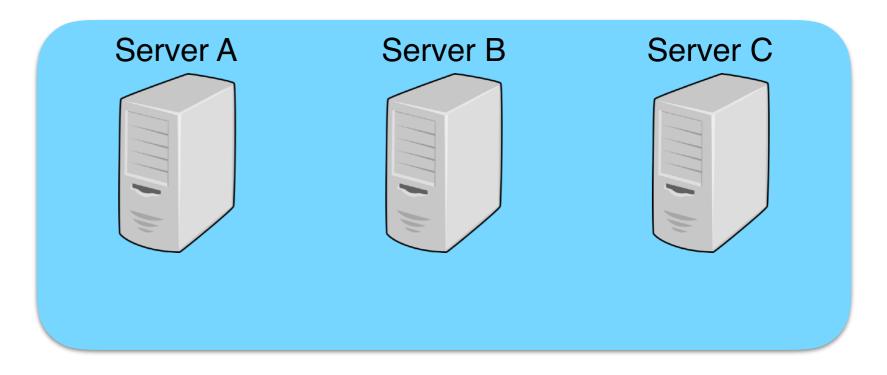
without learning any users' private value x<sub>i</sub>.

**Example:** Privately measuring traffic congestion.

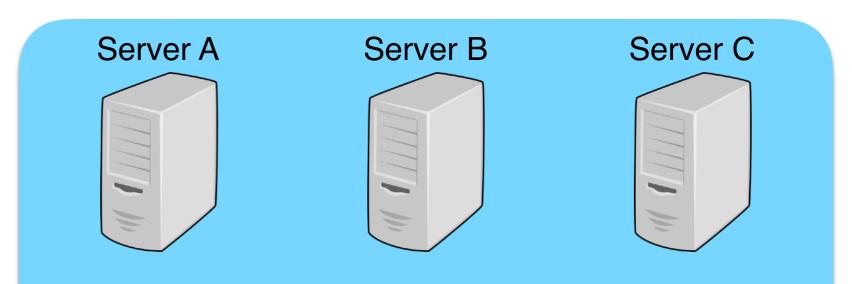


The sum  $x_1 + \ldots + x_N$  yields the number of app users on the Bay Bridge.

[Chaum88], [BGW88], ... [KDK11] [DFKZ13] [PrivEx14] ...



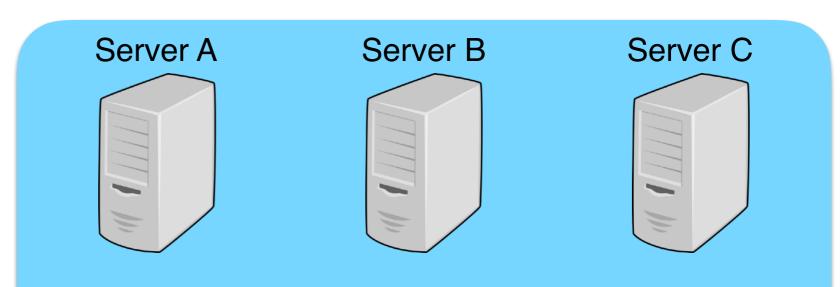
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Assume that the servers are non-colluding.

Equivalently: that at least one server is honest.

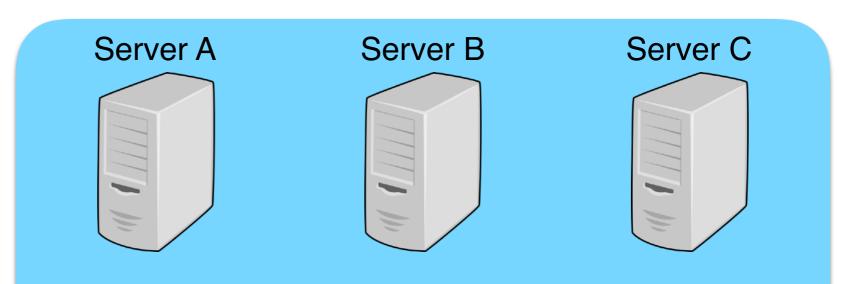
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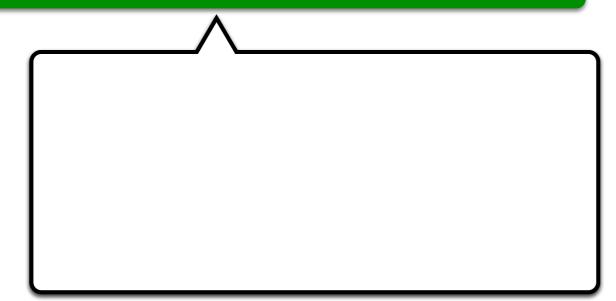


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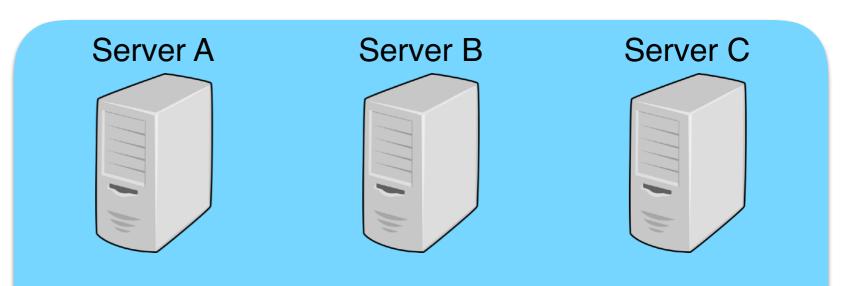


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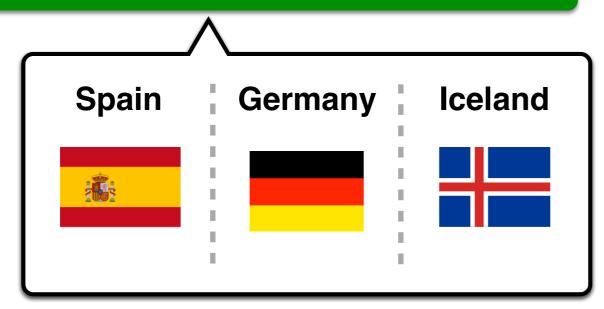


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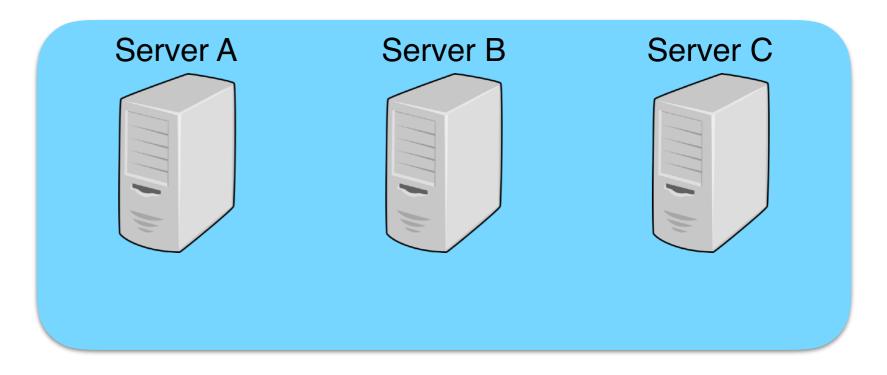


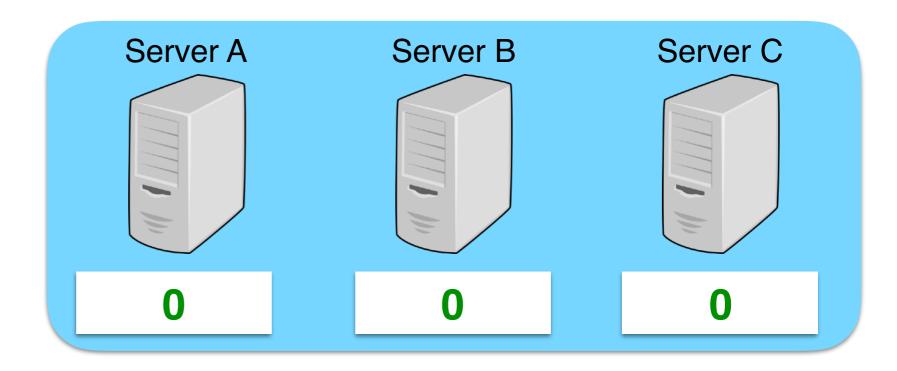
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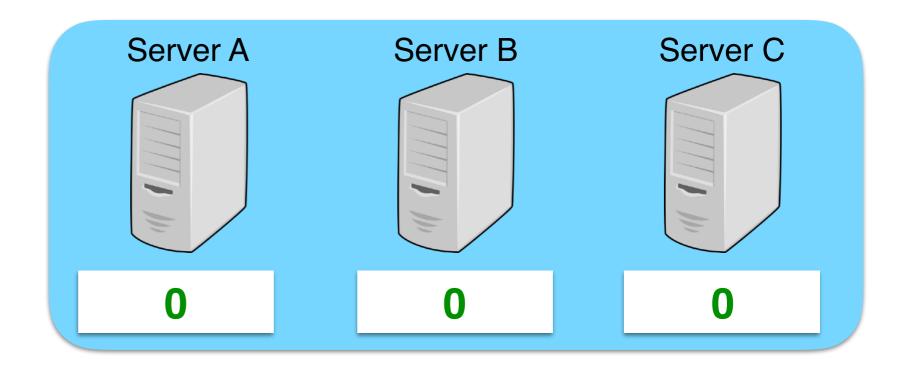


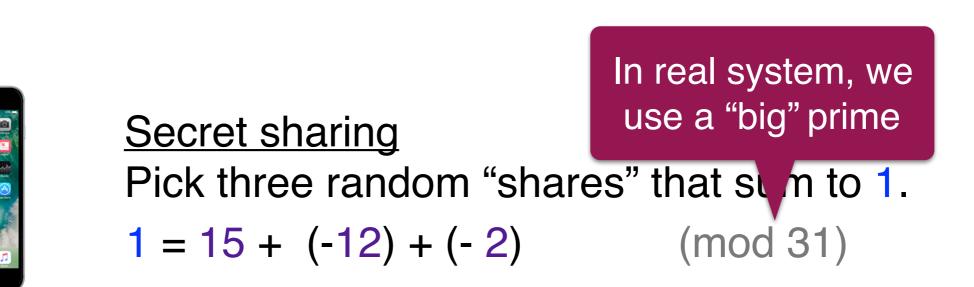


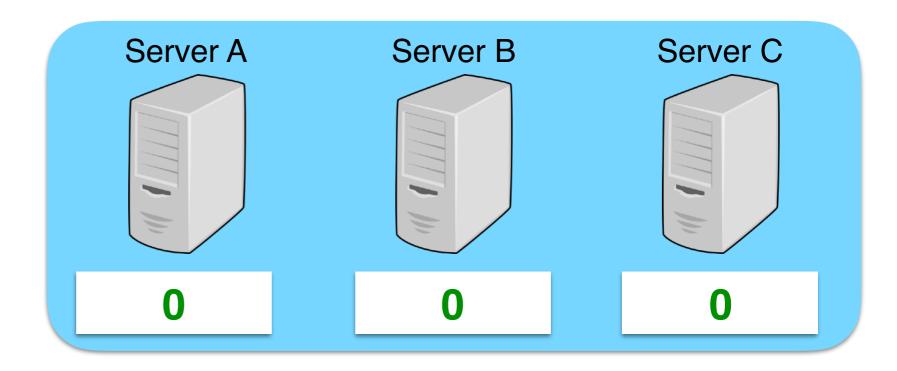
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Secret sharing Pick three random "shares" that sum to 1. 1 = 15 + (-12) + (-2) (mod 31)



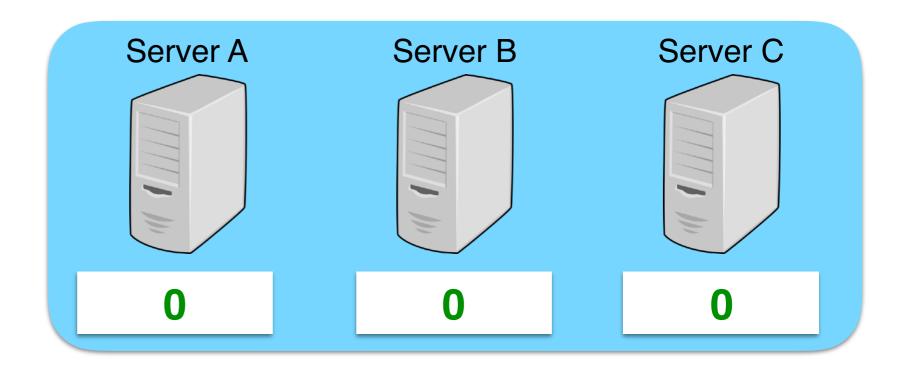




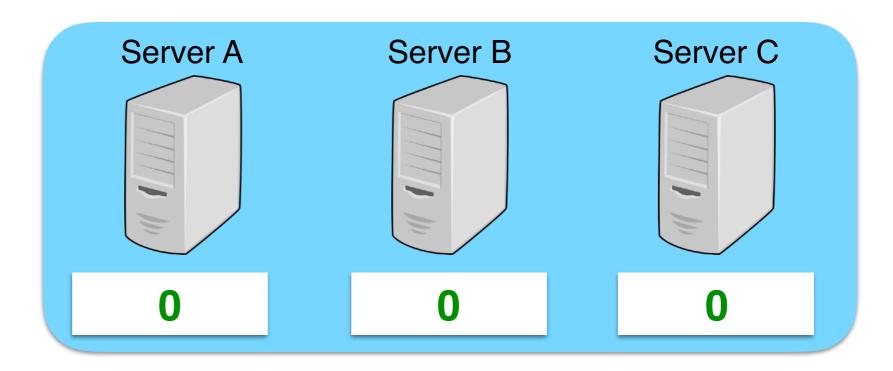
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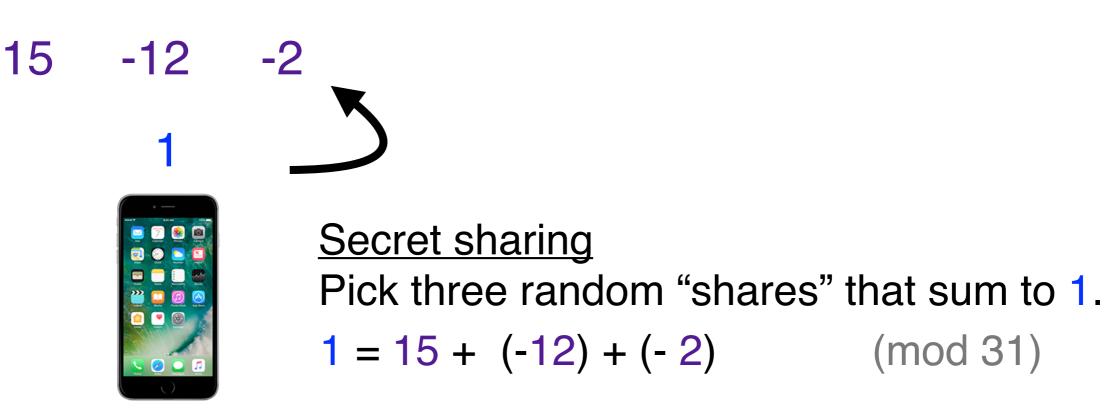


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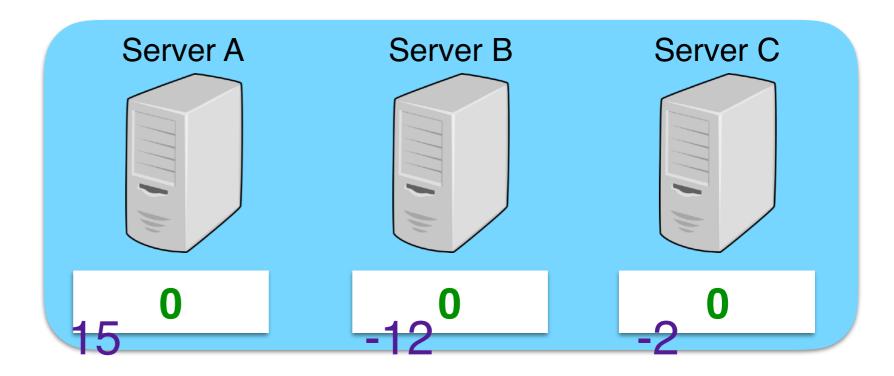


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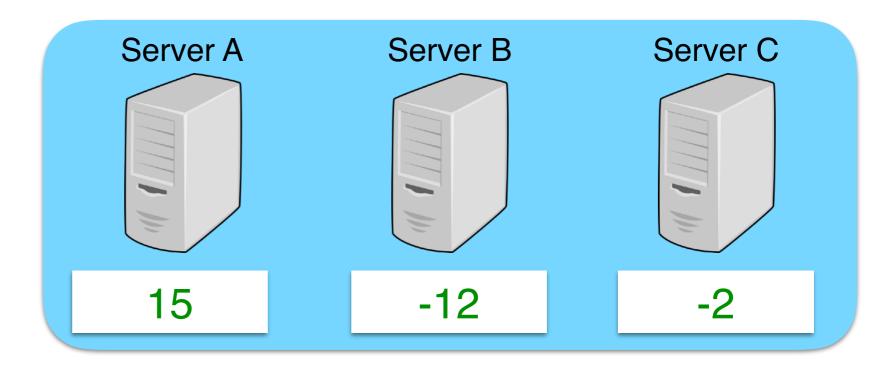




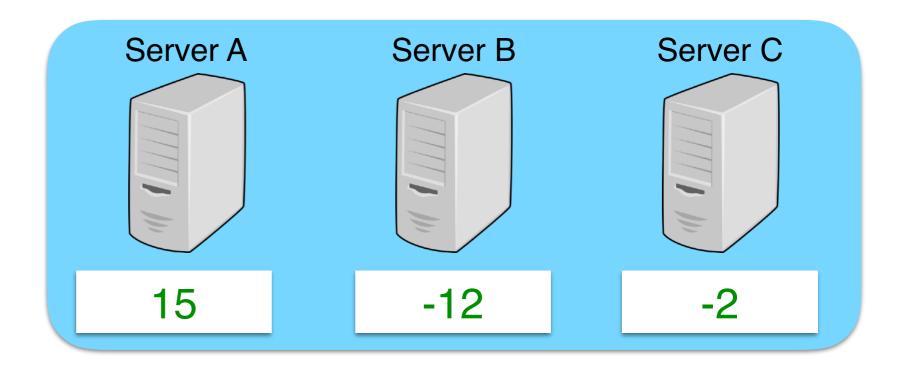
Need all three shares to recover the shared value.







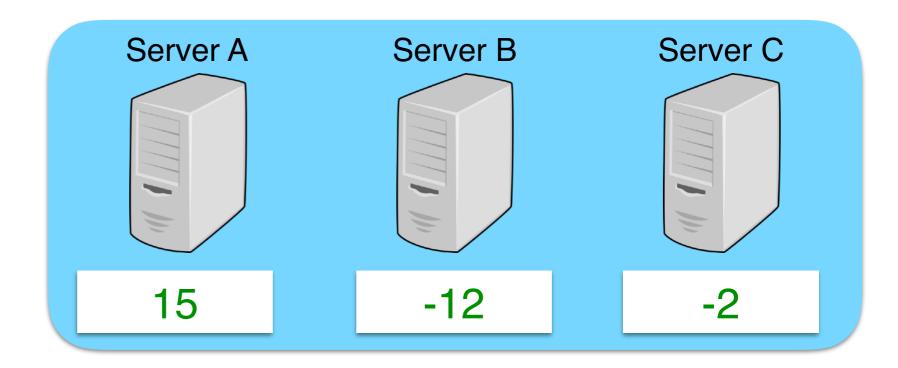








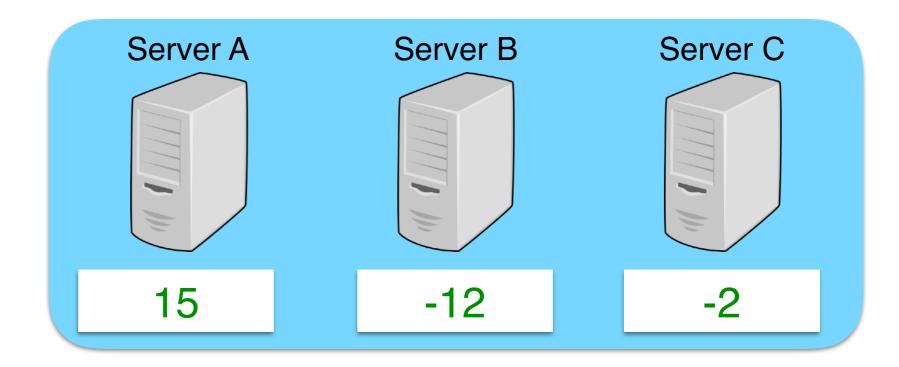


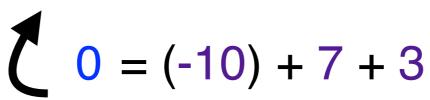


0 = (-10) + 7 + 3



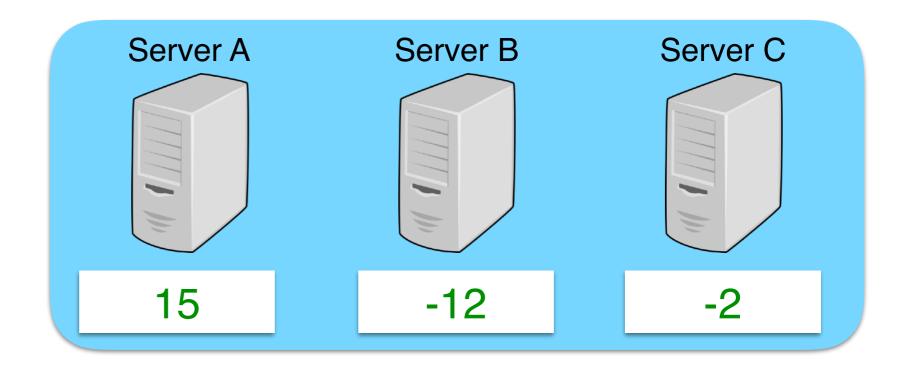






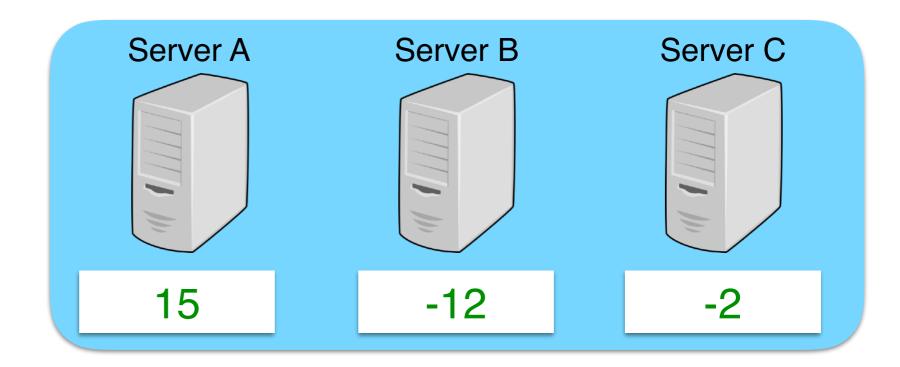




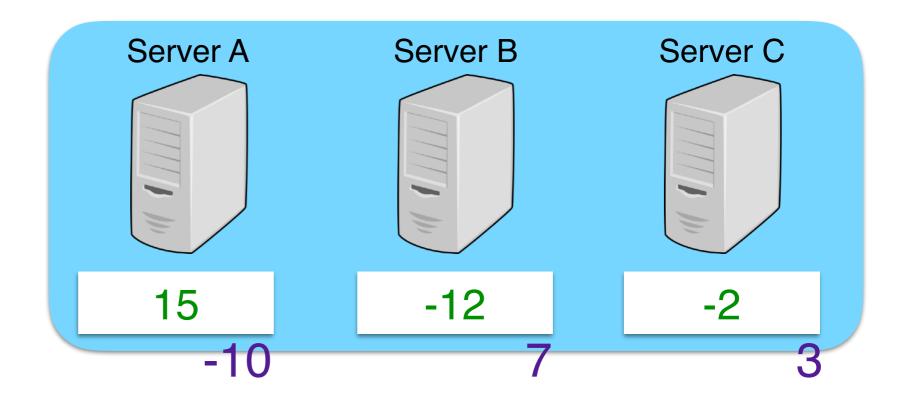


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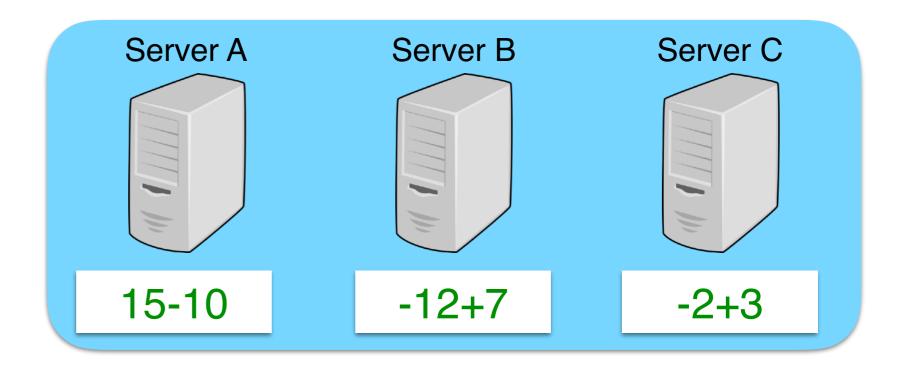












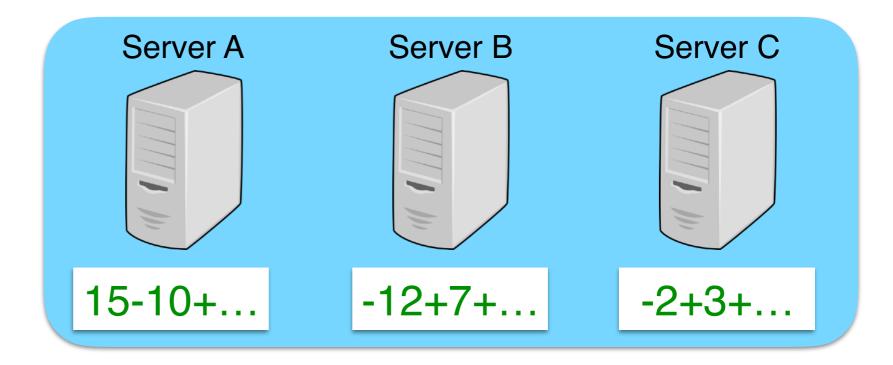


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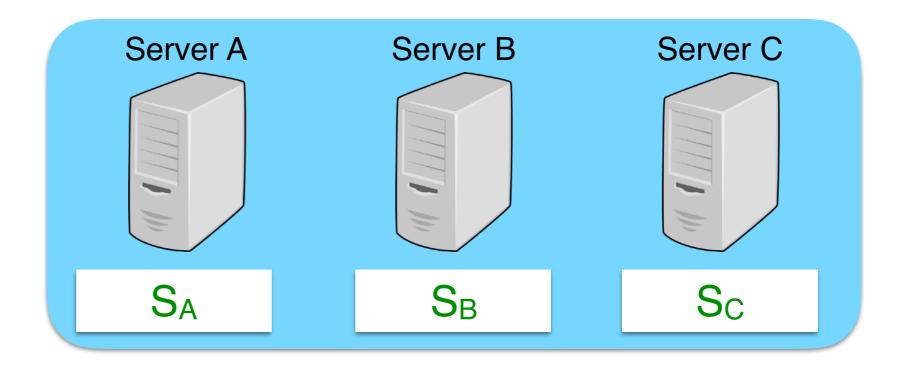


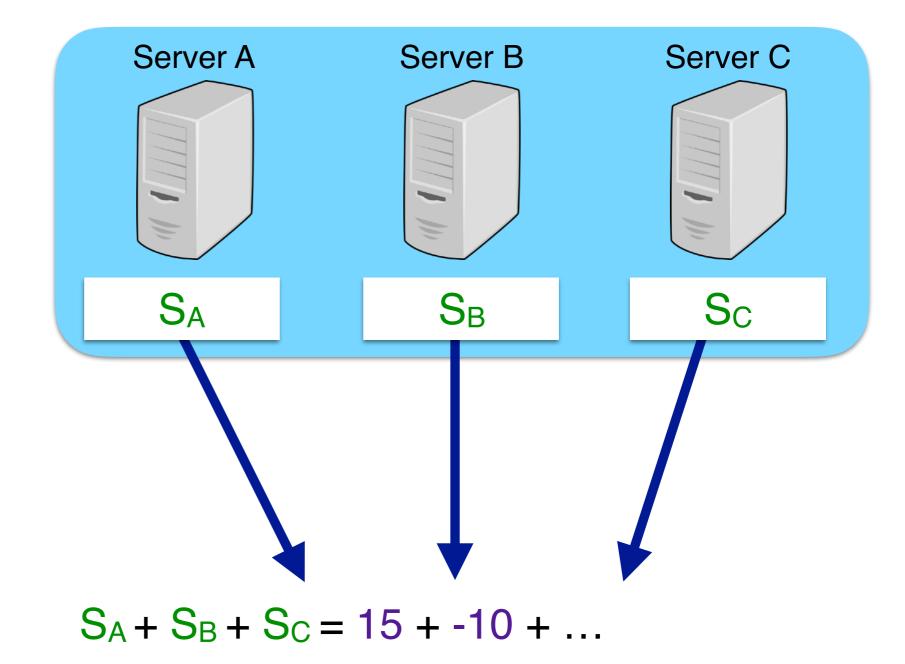


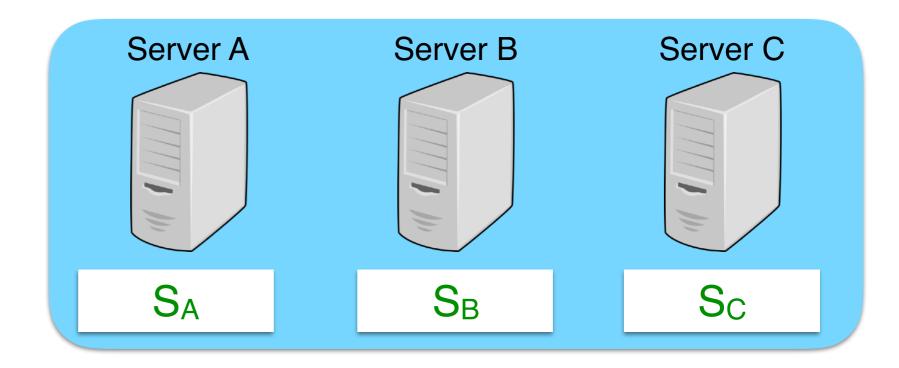




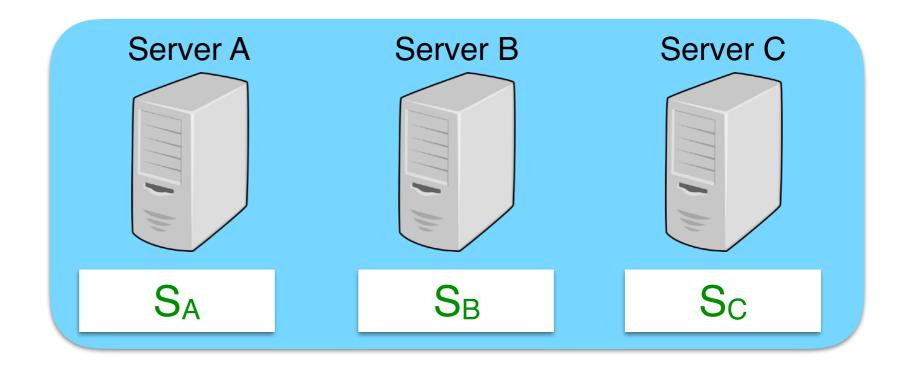




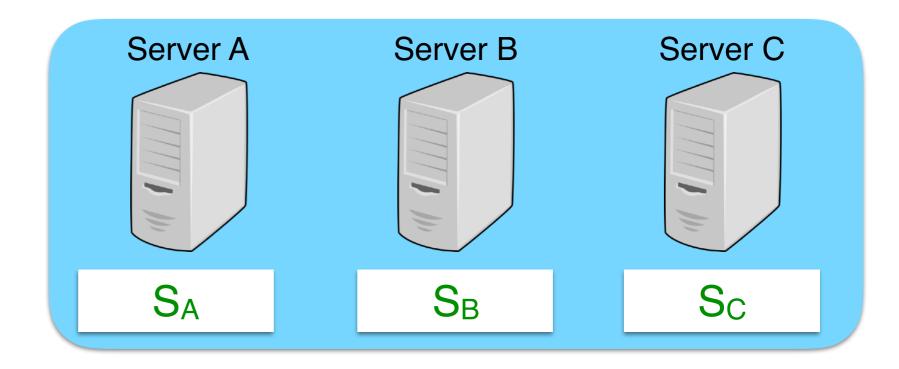


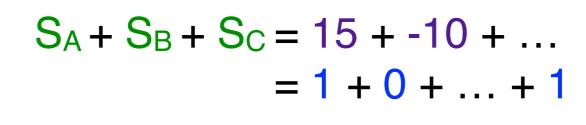


$$S_A + S_B + S_C = 15 + -10 + \dots$$

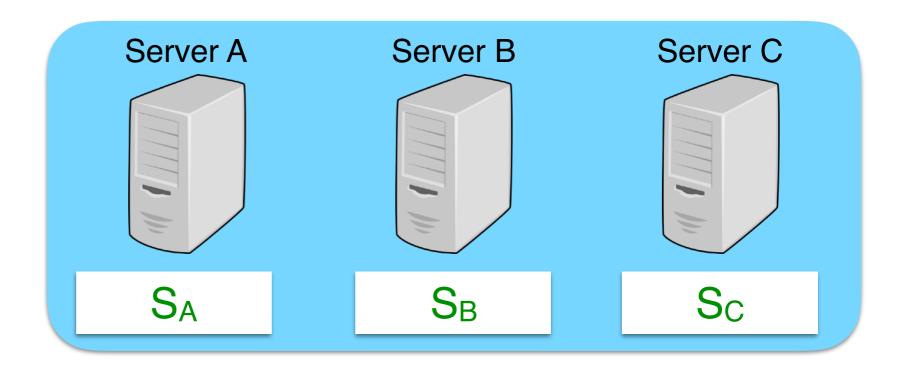


$$S_A + S_B + S_C = 15 + -10 + \dots$$
  
= 1 + 0 + \dots + 1





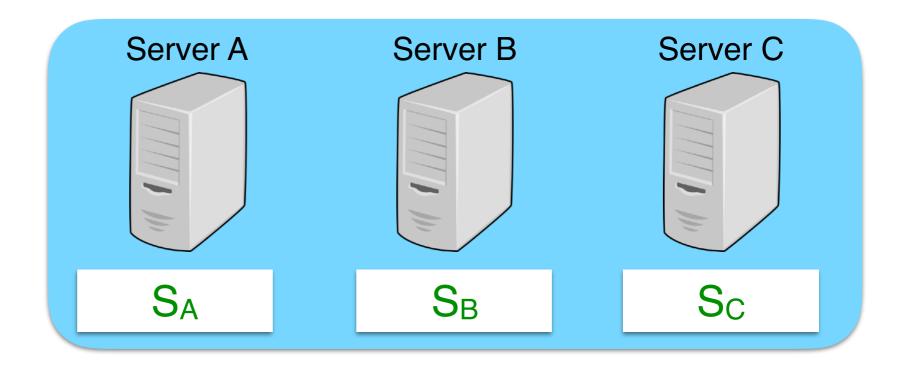
Servers learn the sum of client values and learn *nothing else*.





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 $S_A + S_B + S_C = 15 + -10 + \dots$ = 1 + 0 + \dots + 1

> Learn that three phones are on the Bay Bridgedon't know which three

### Computing private sums

# Computing private sums

**Exact correctness:** If everyone follows the protocol, servers compute the sum of all  $x_i$ s.

**Privacy:** Any proper subset of the servers learns nothing but the sum of the  $x_i$ s.

Efficiency: Follows by inspection.

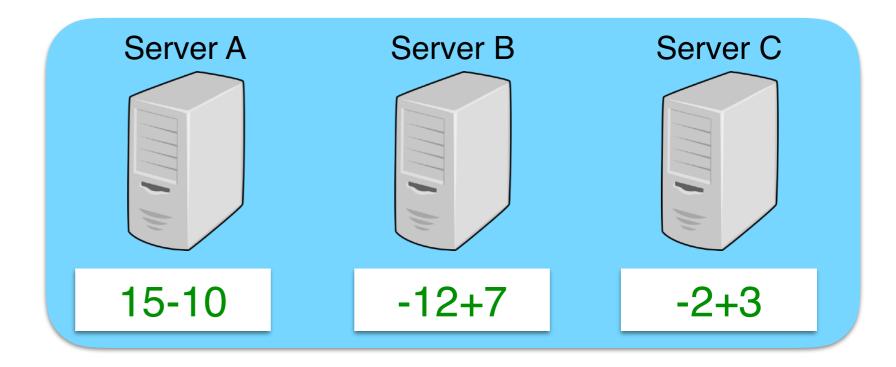
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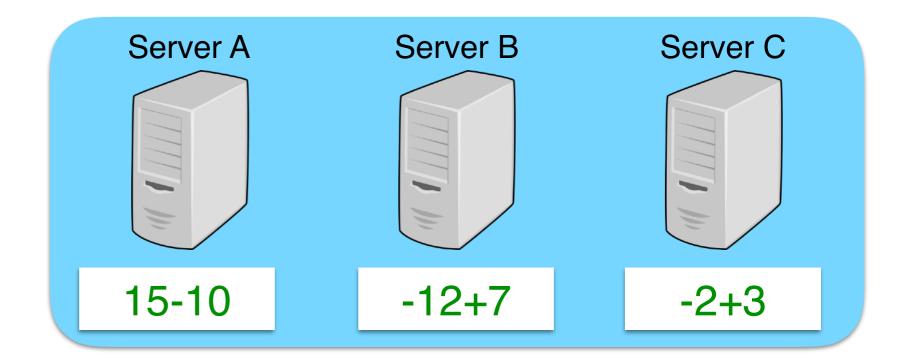




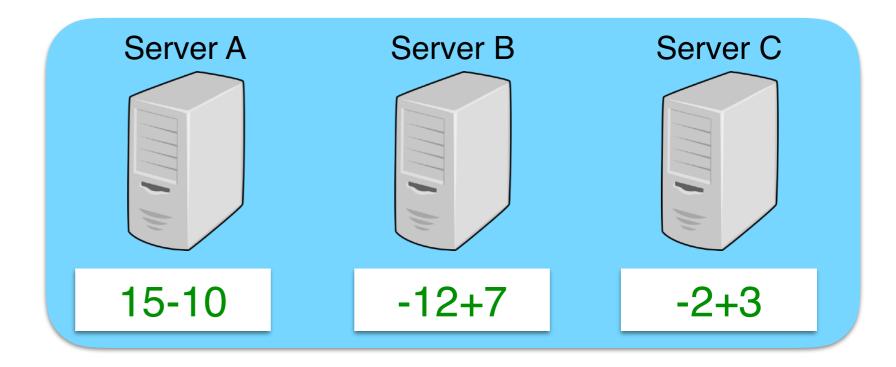










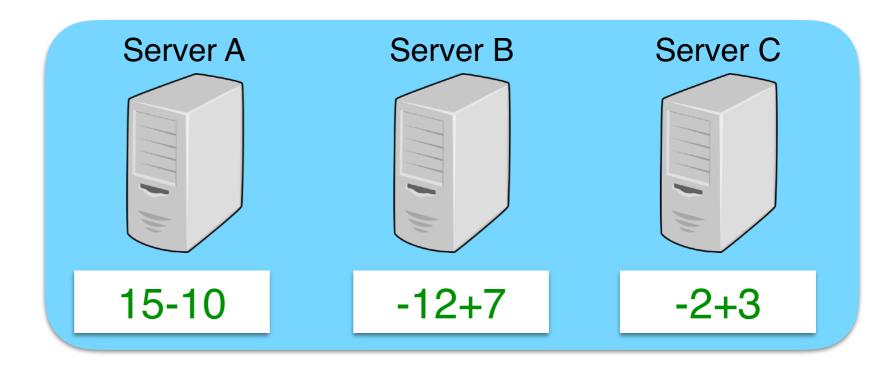








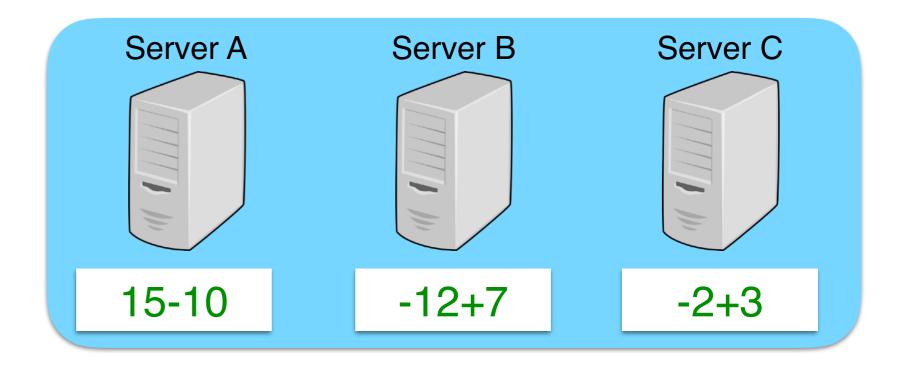










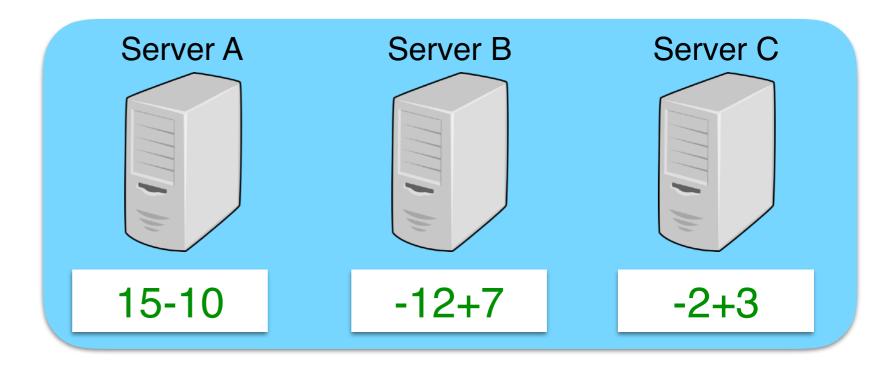


An evil client needn't follow the rules!

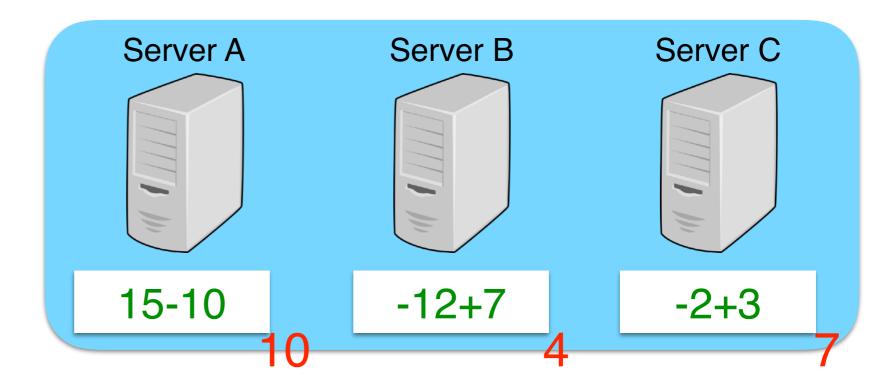
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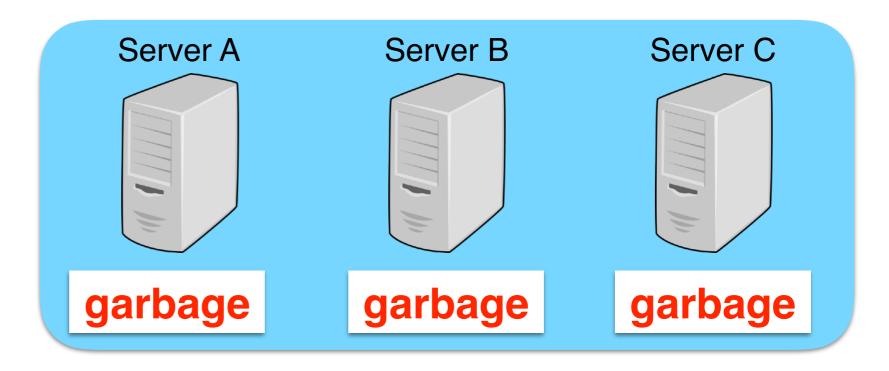








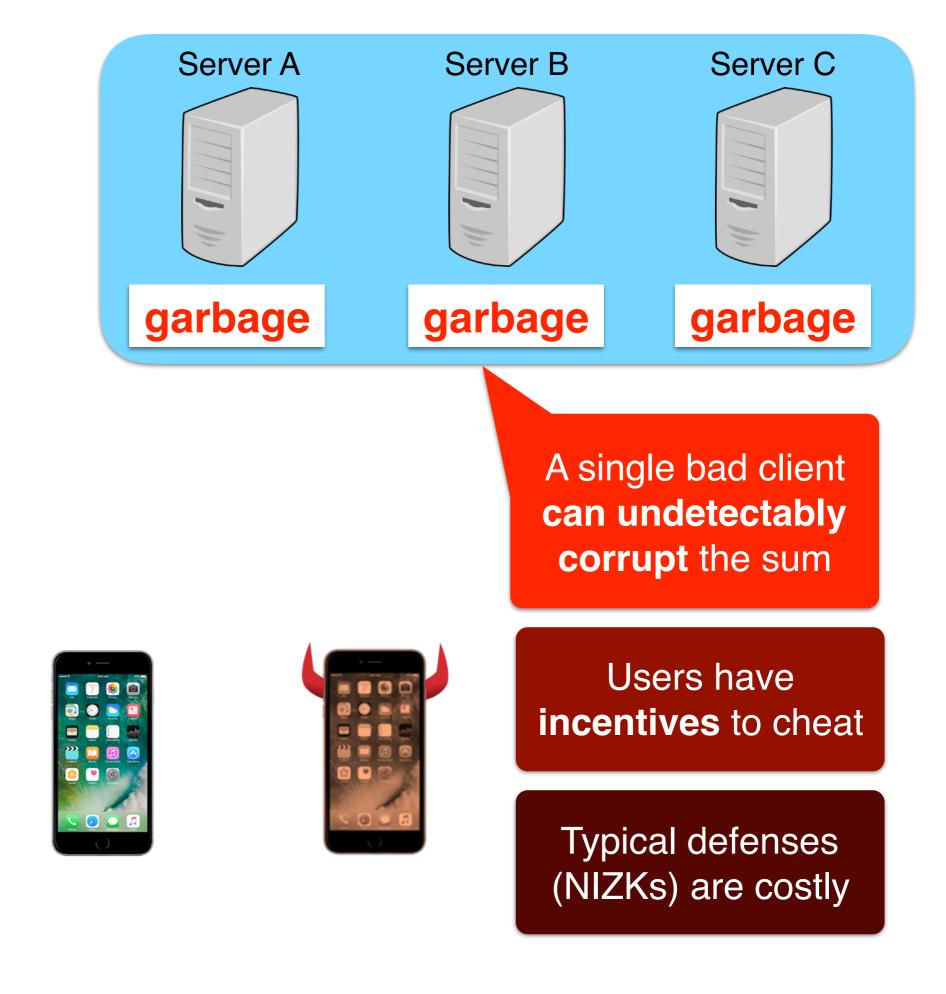












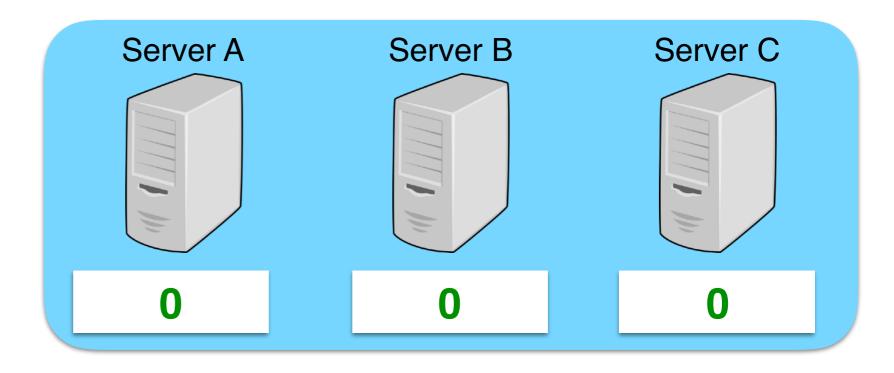


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- Providing robustness with SNIPs
- Evaluation
- Encodings for <u>complex</u> aggregates

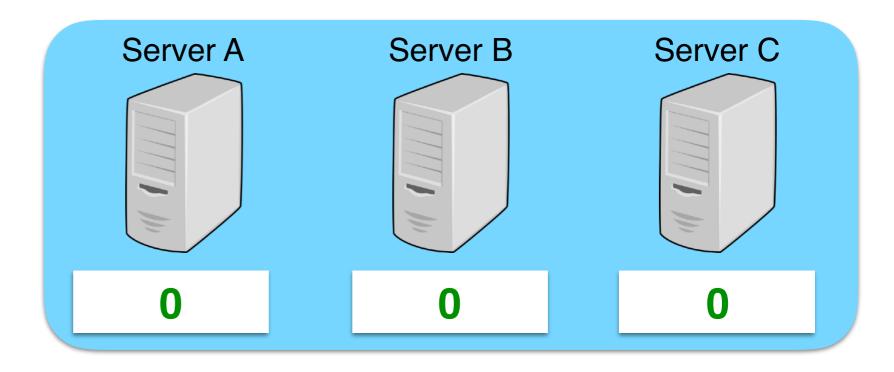
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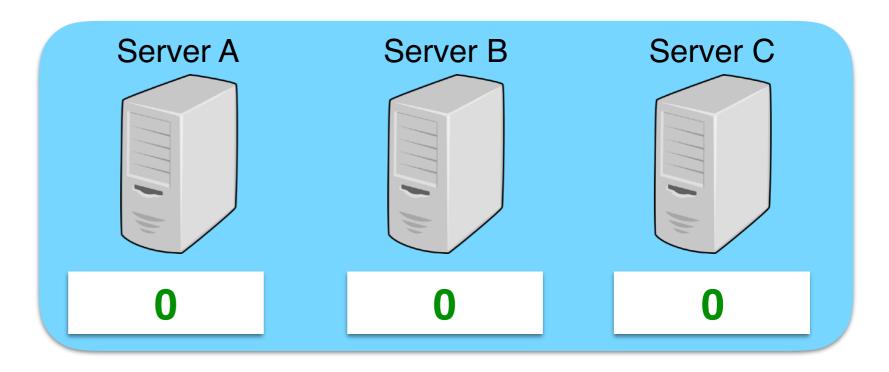


x = 1

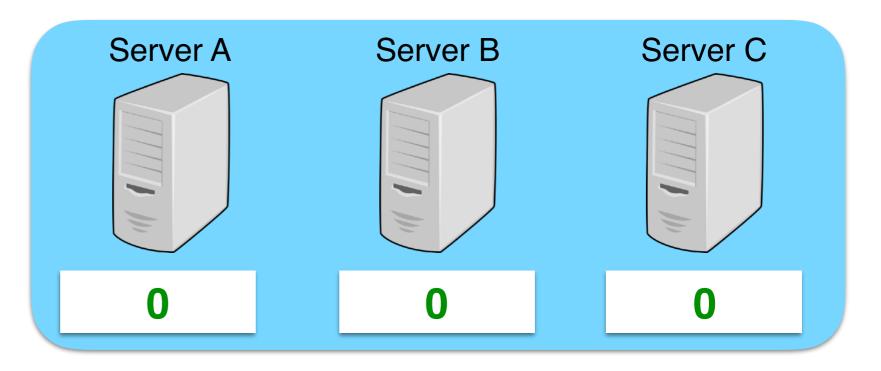


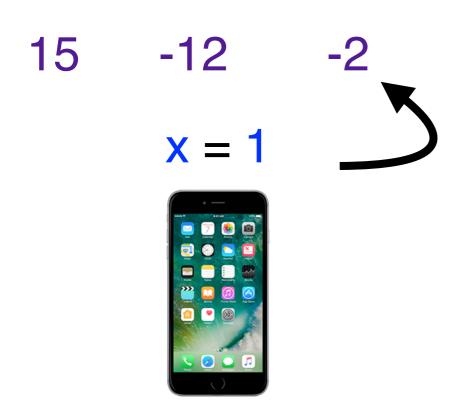


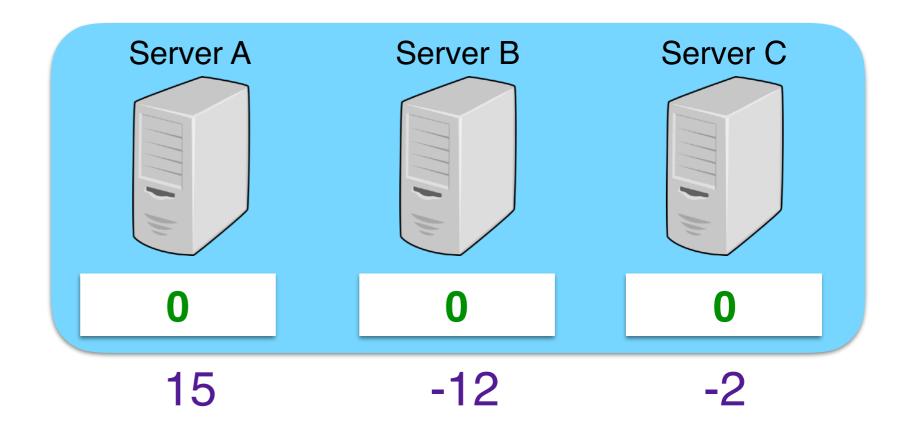




15 + (-12) + (-2) = 1x = 1

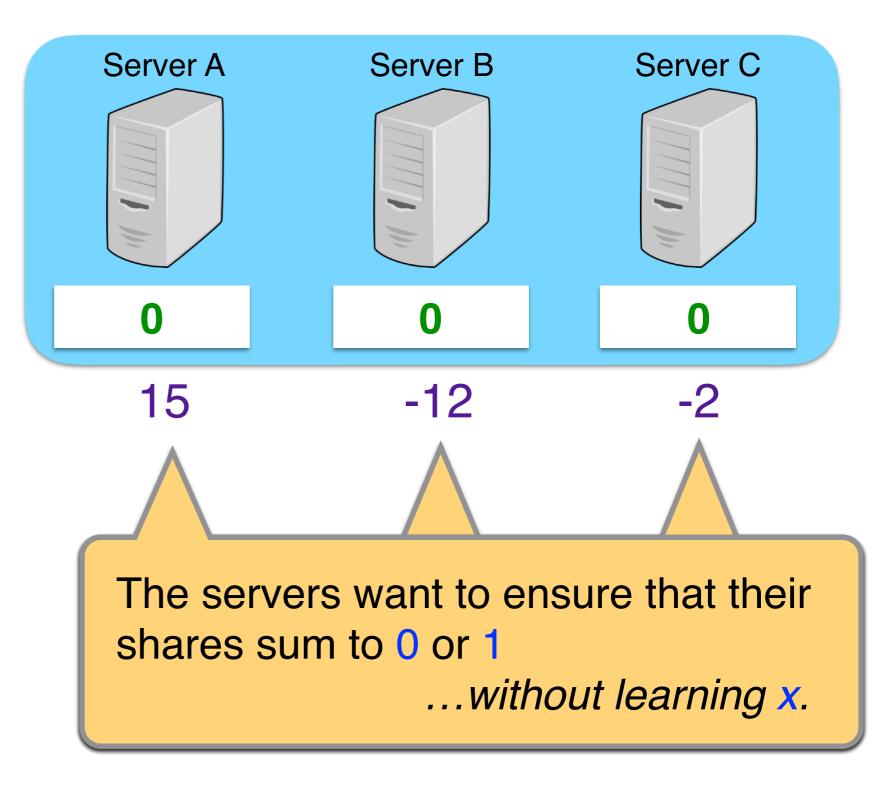


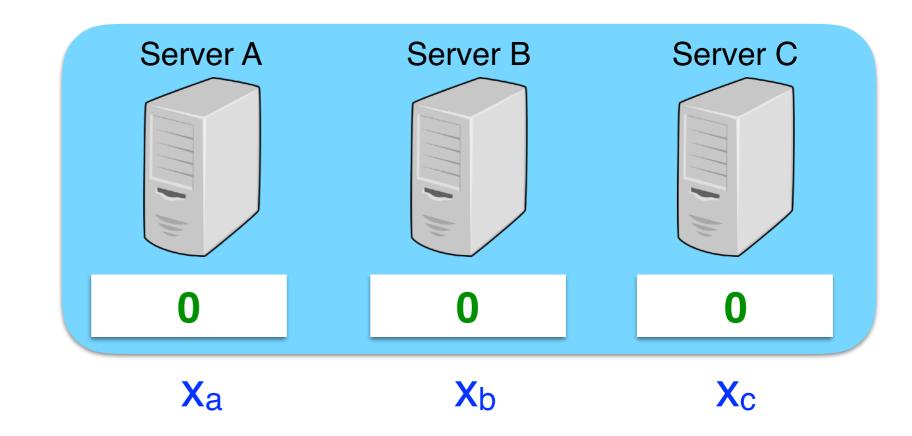










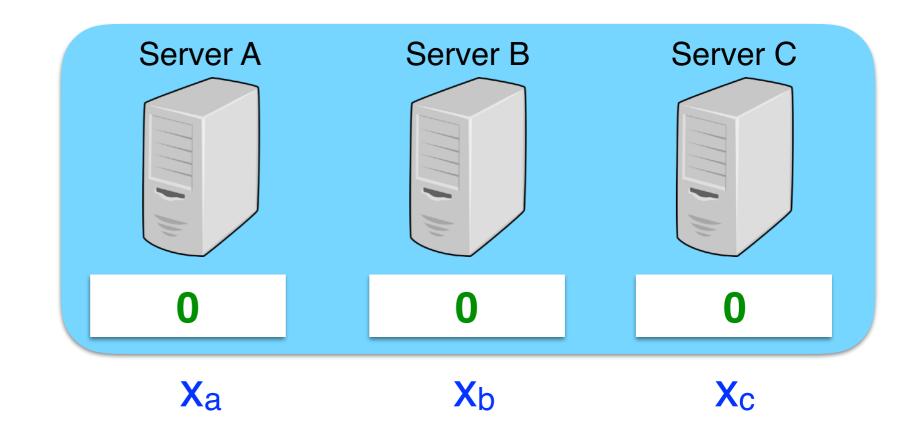


x = 1



#### More generally, servers

- hold shares of the client's private value x
- hold an <u>arbitrary</u> public predicate Valid(·)
   expressed as an arithmetic circuit
- want to test if "Valid(x)" holds, without leaking x



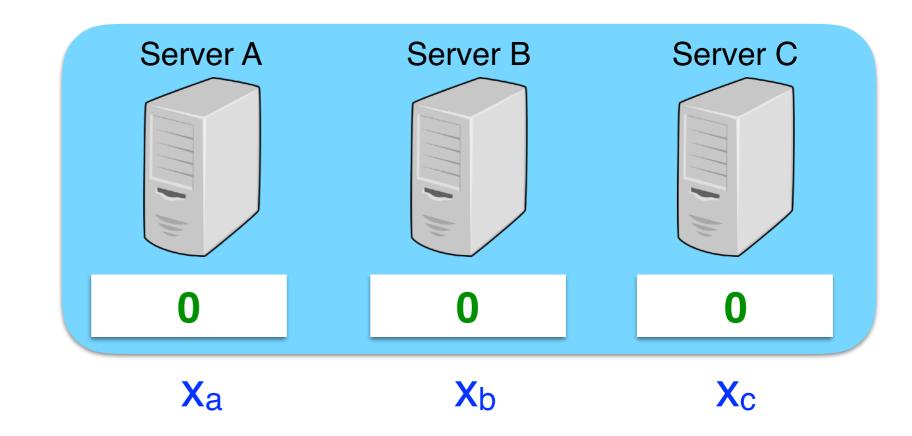




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Valid(x) = " $x \in \{0,1\}$ "

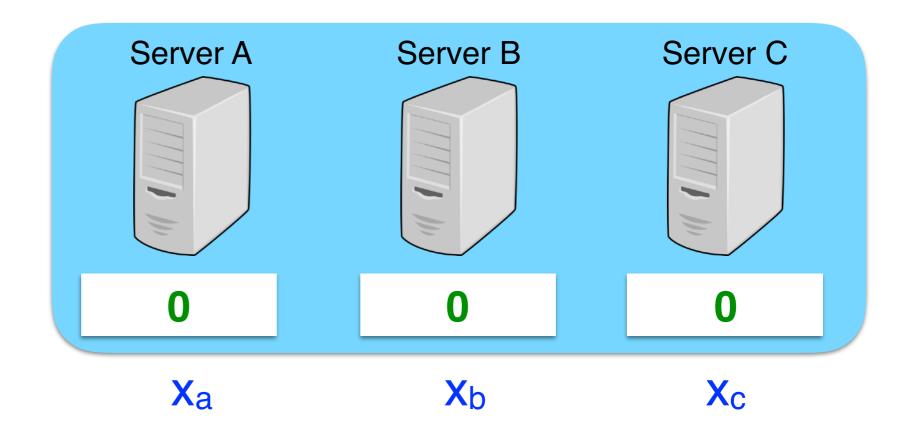


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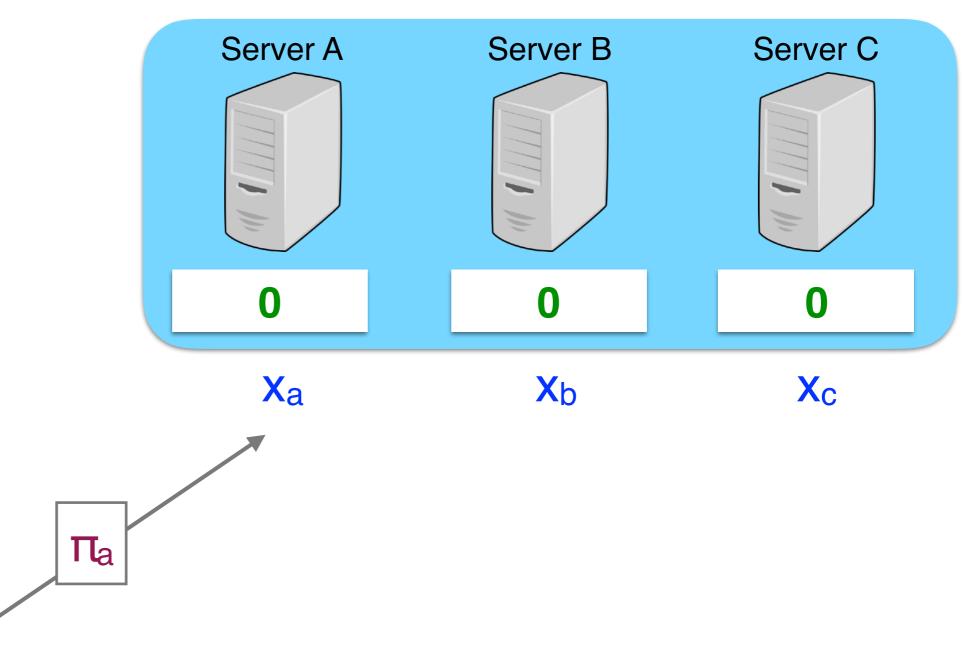


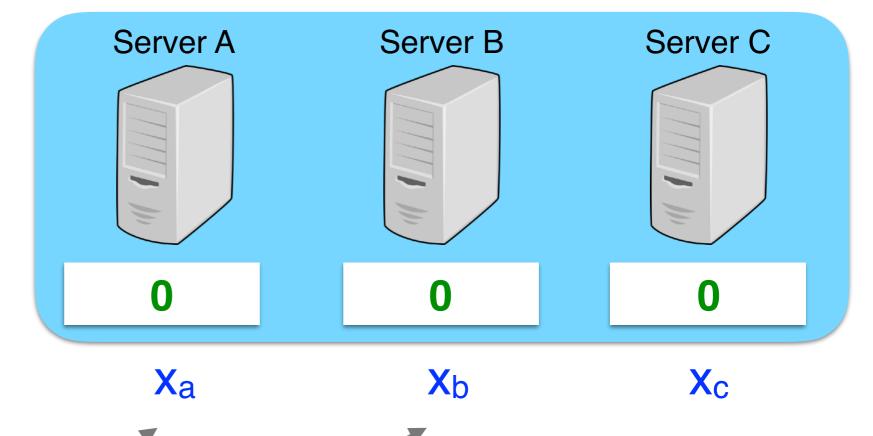


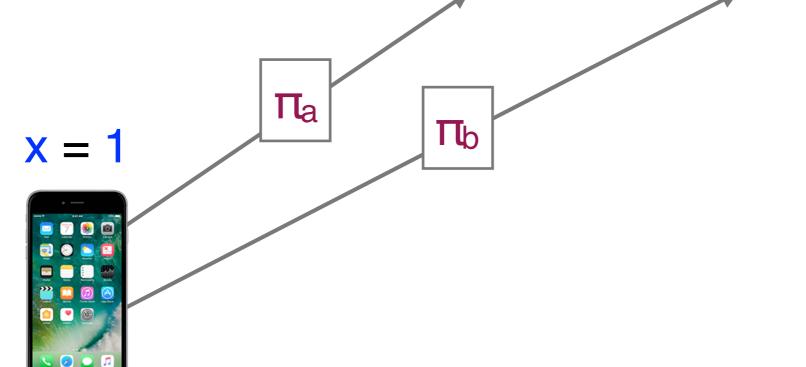
 $\mathbf{x} = \mathbf{1}$ 

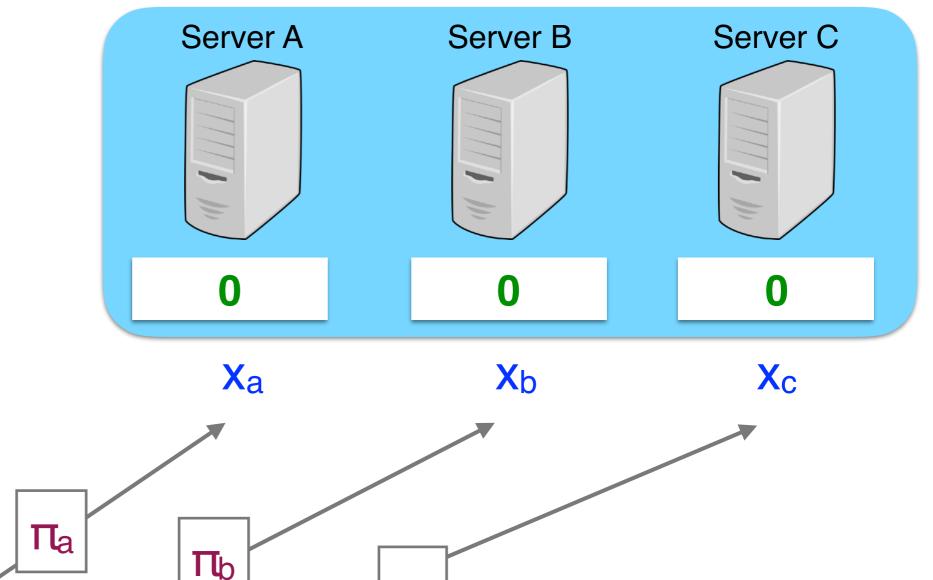
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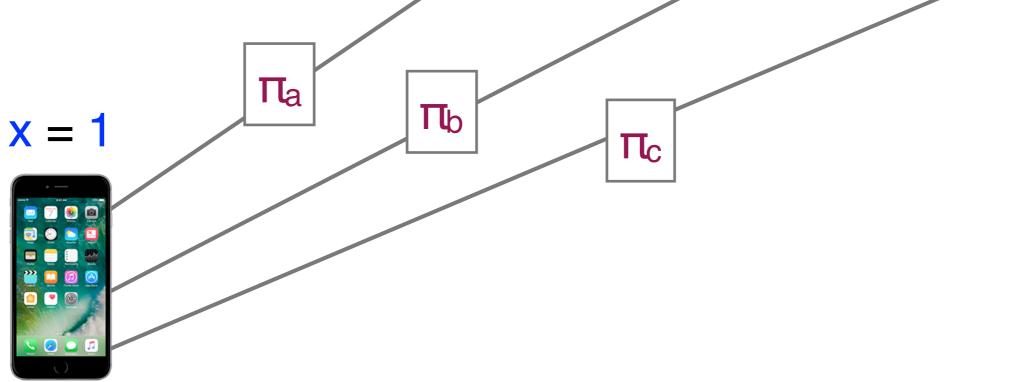
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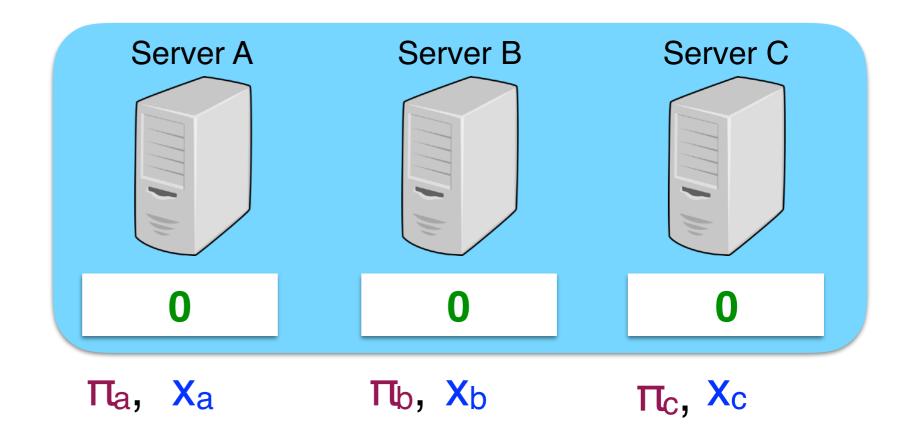


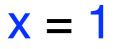




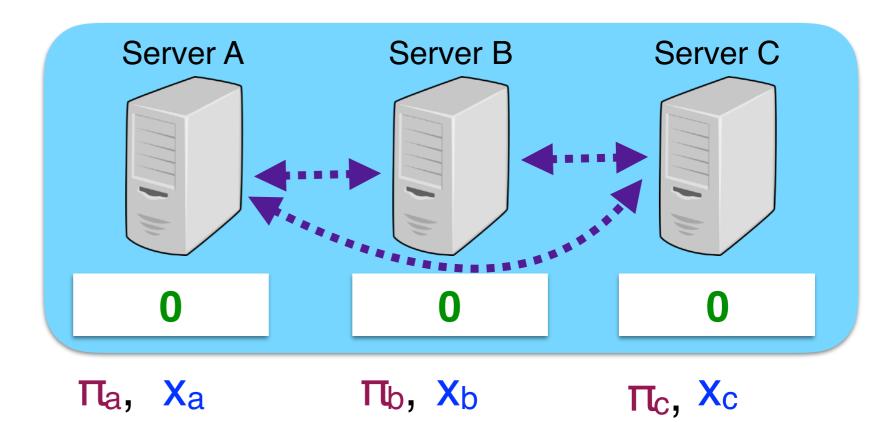




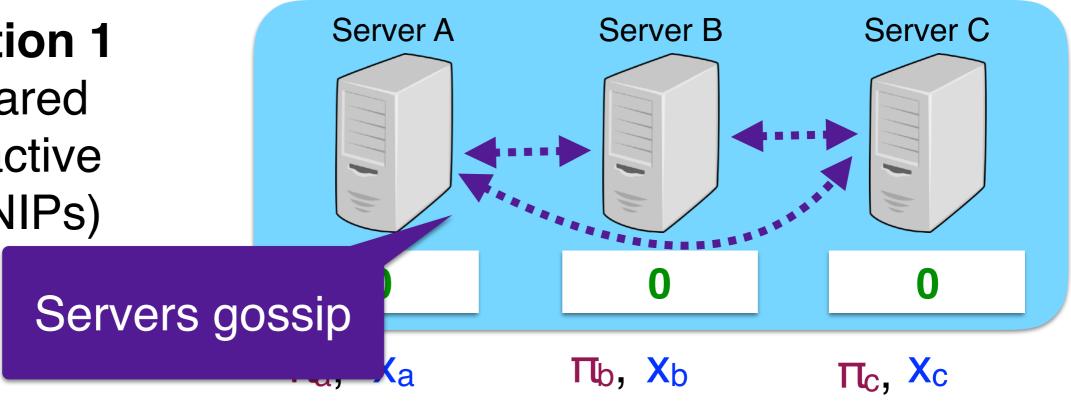






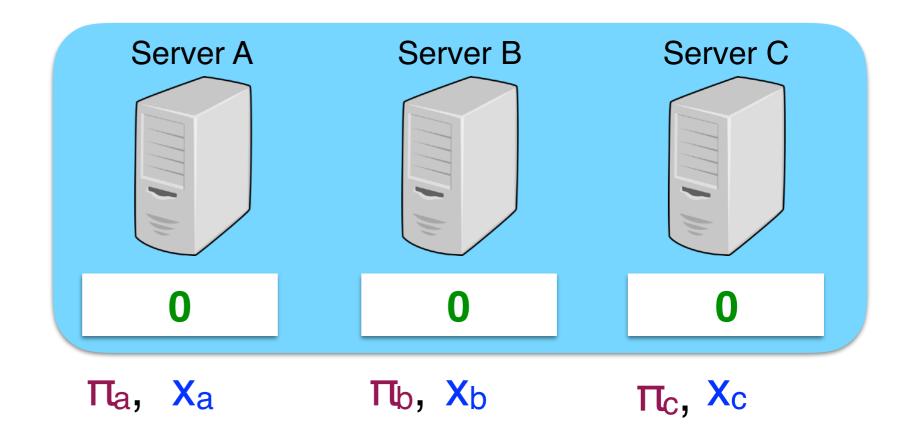


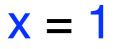




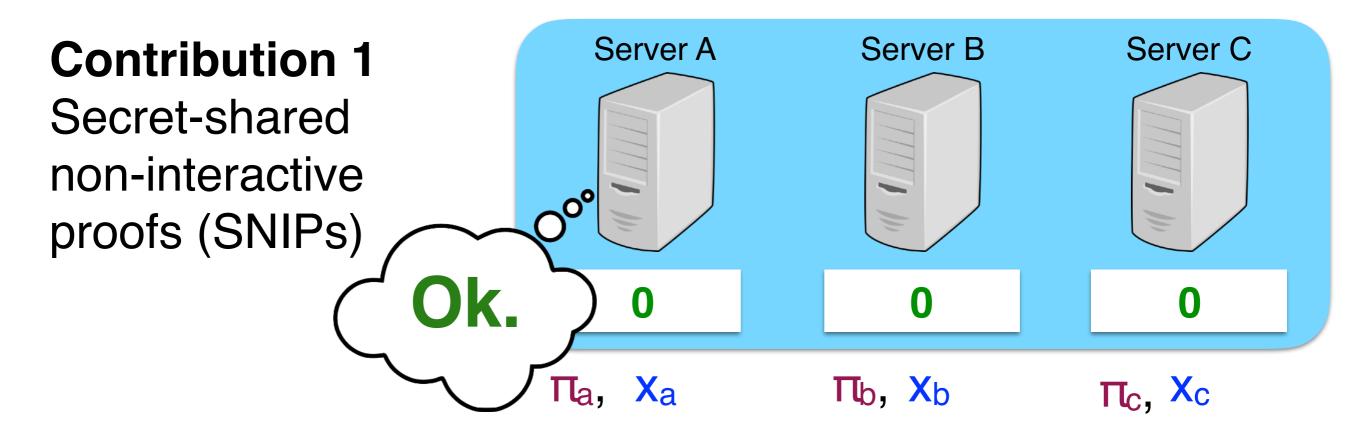




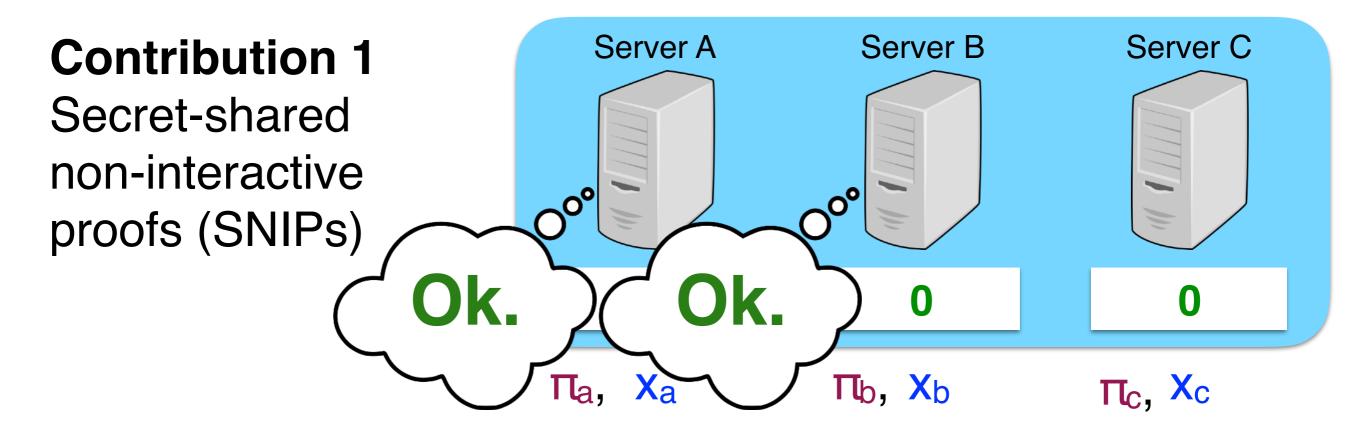




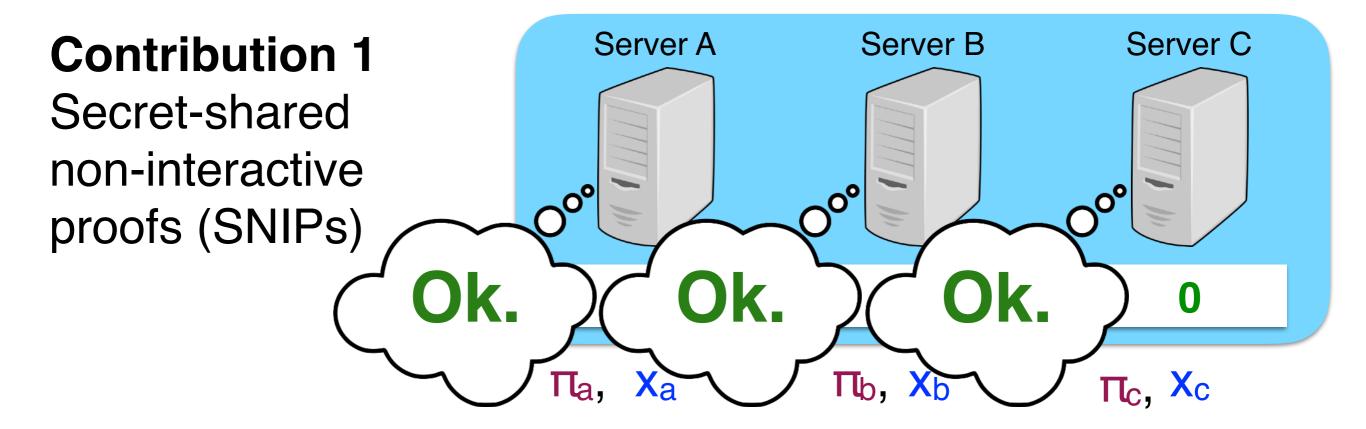




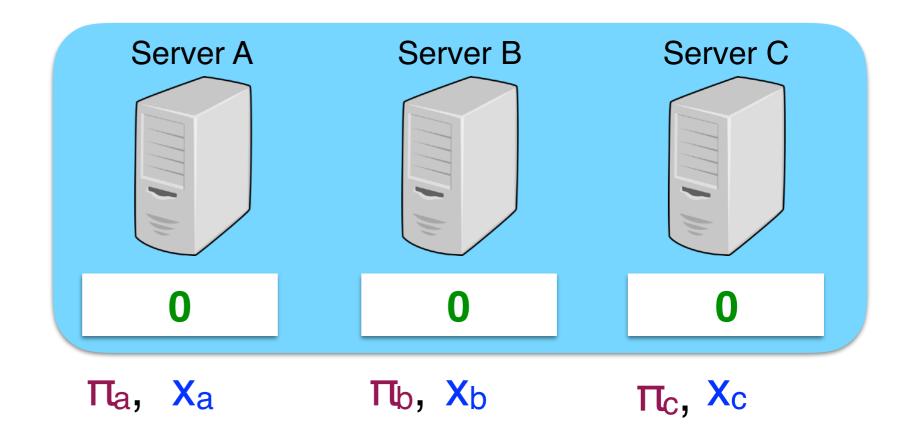


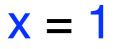




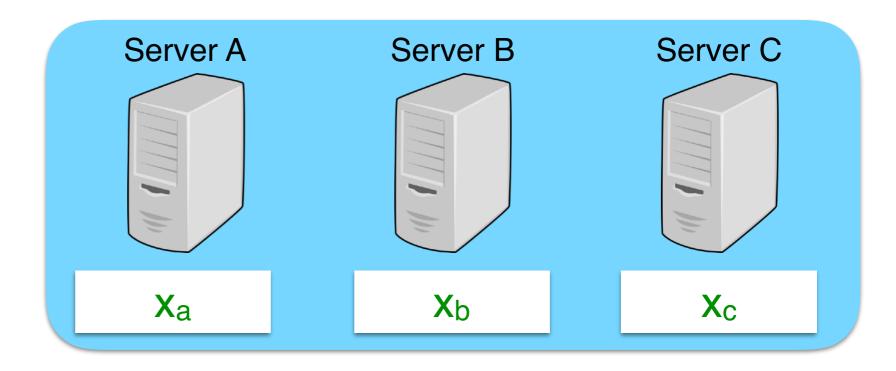




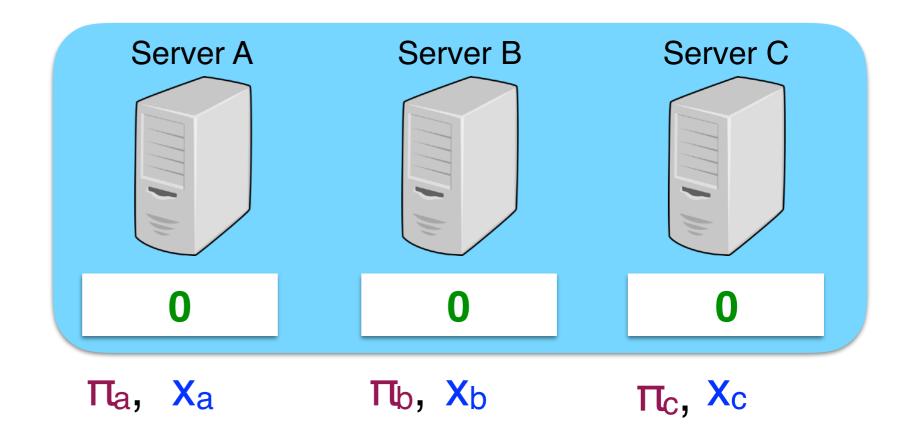


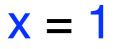




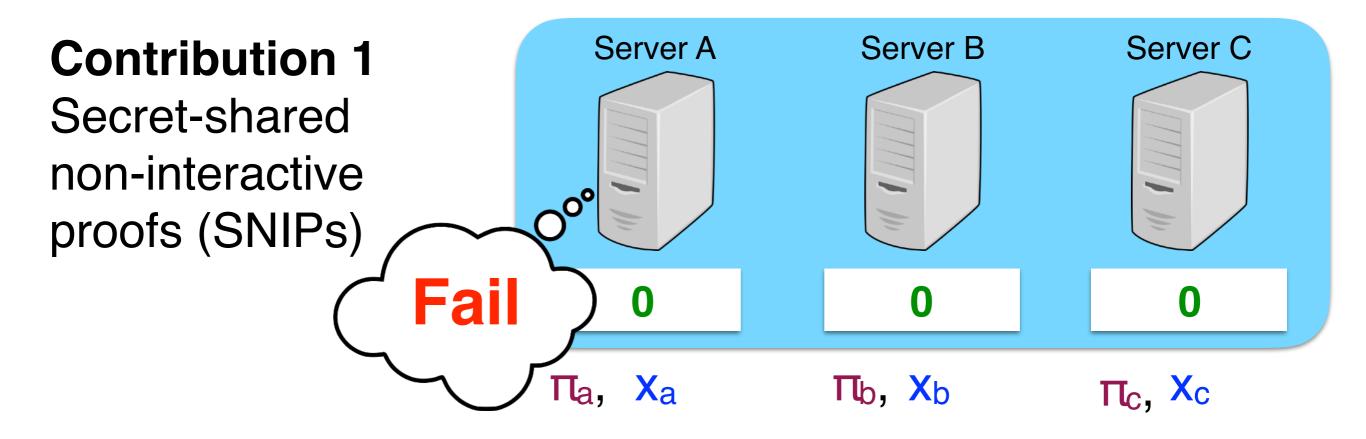




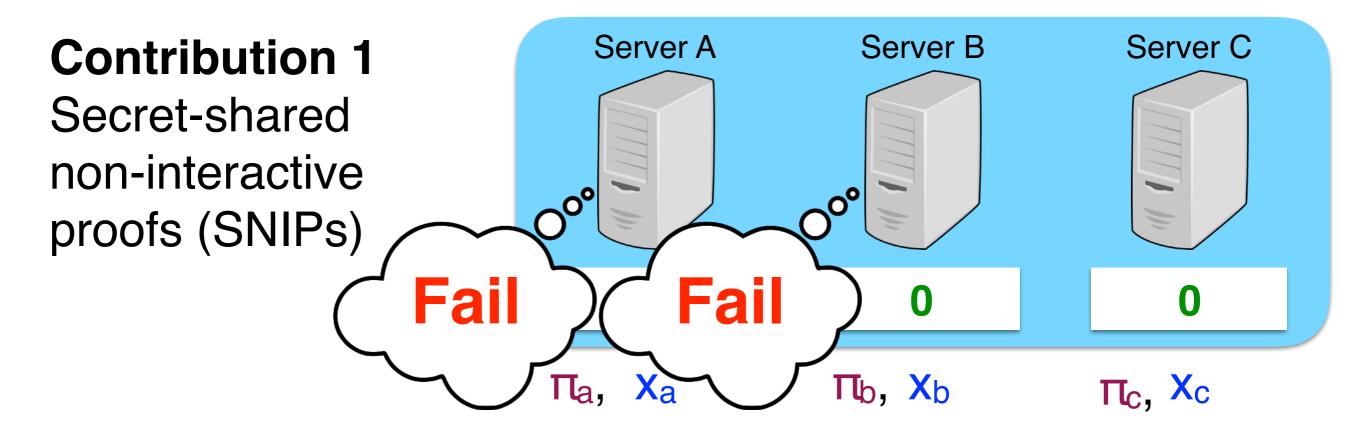




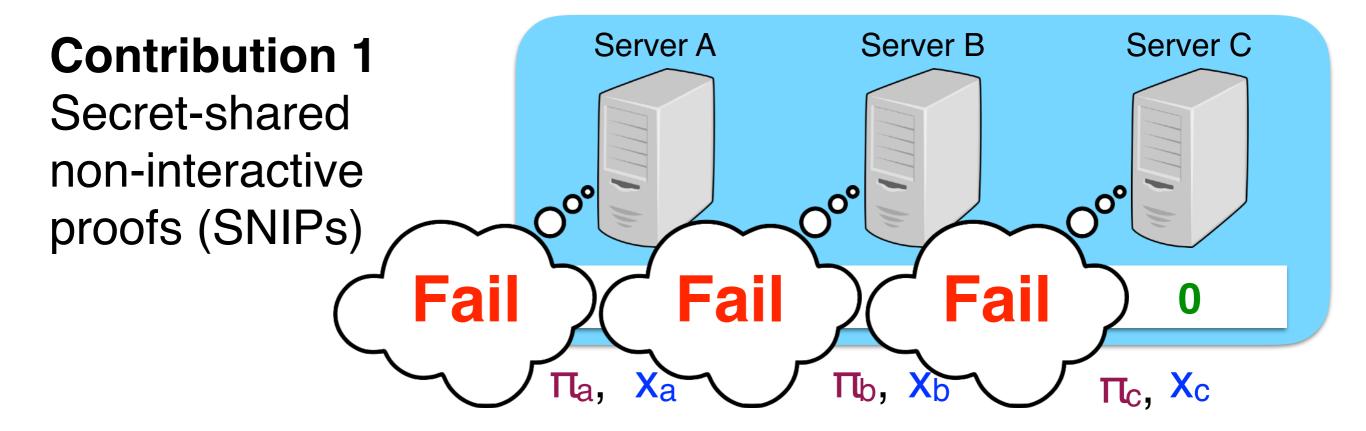




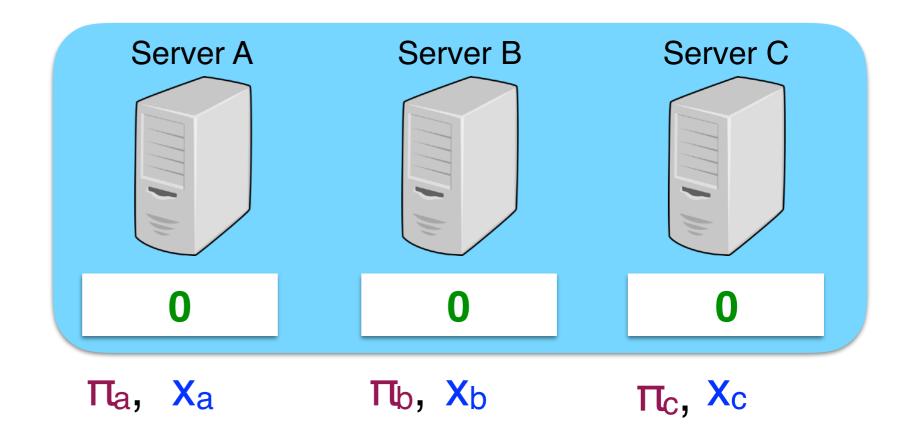


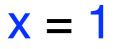




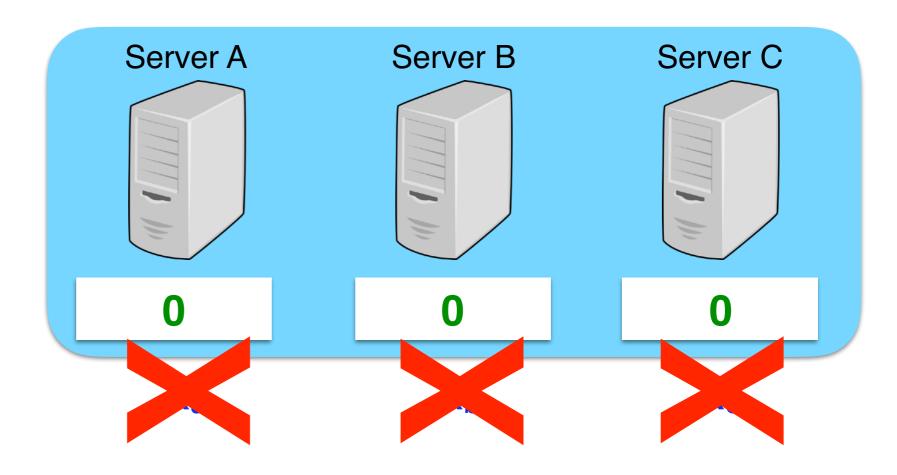












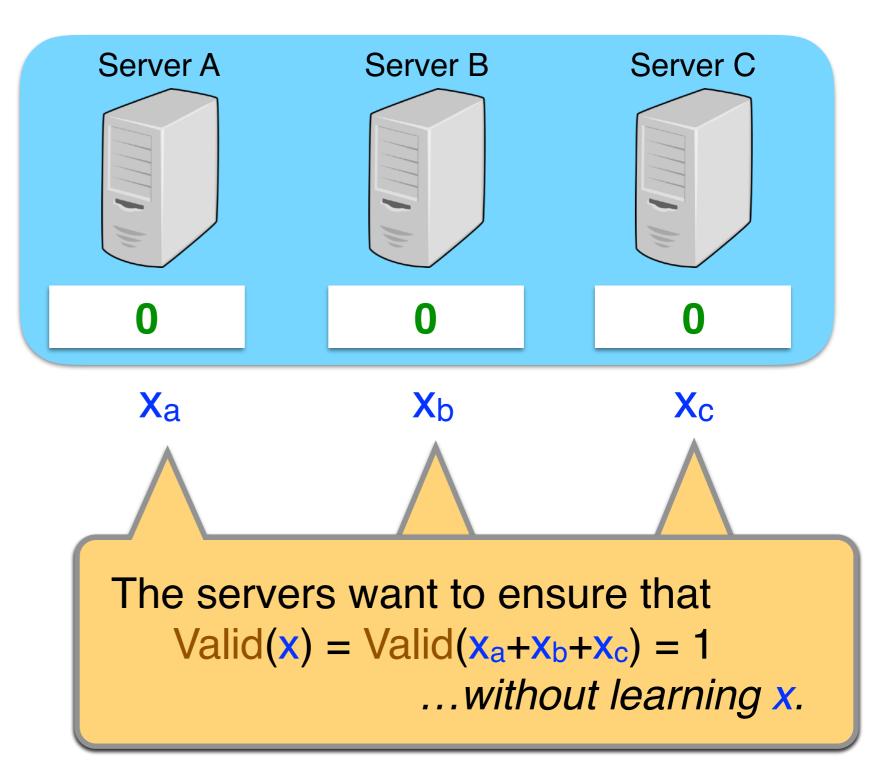


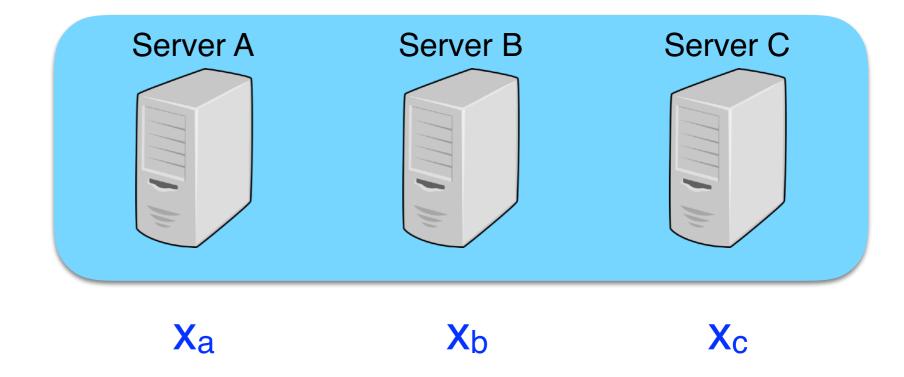


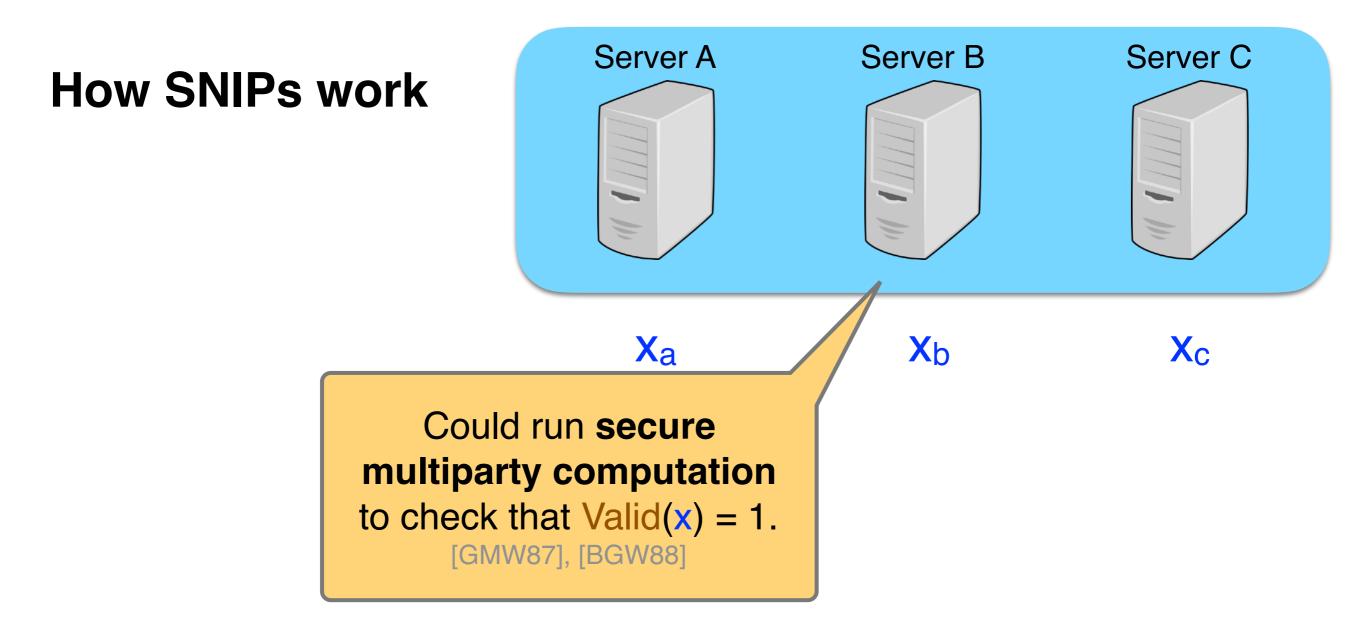
- Prio servers detect and reject malformed client submissions
- In this example, each client can influence the aggregate statistic by +/- 1, at most

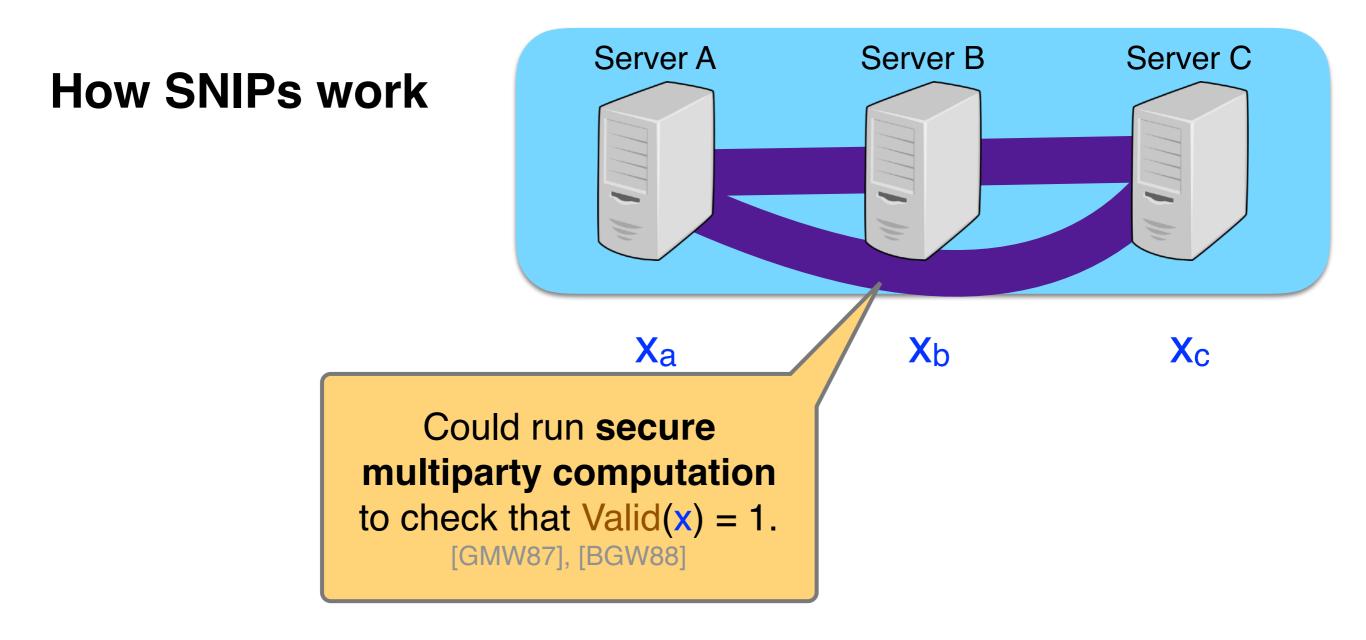


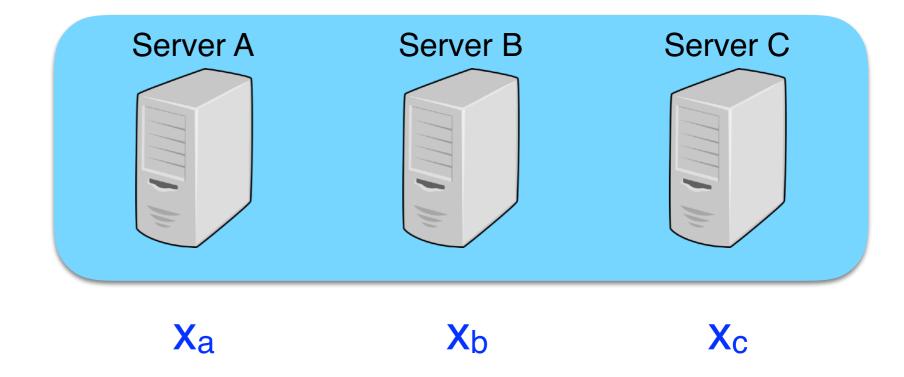








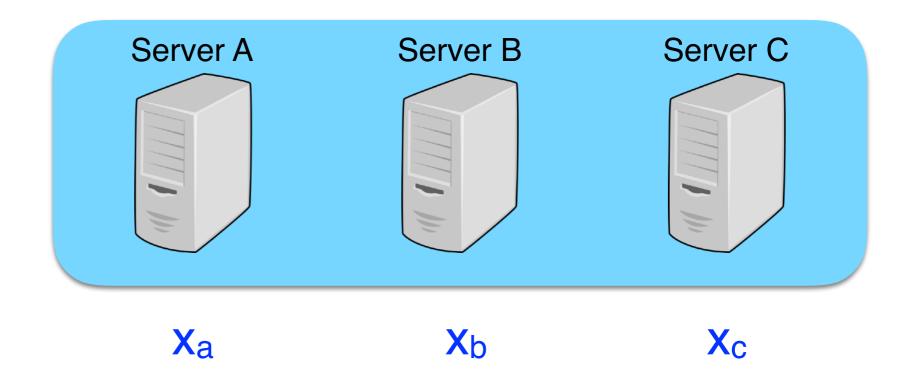


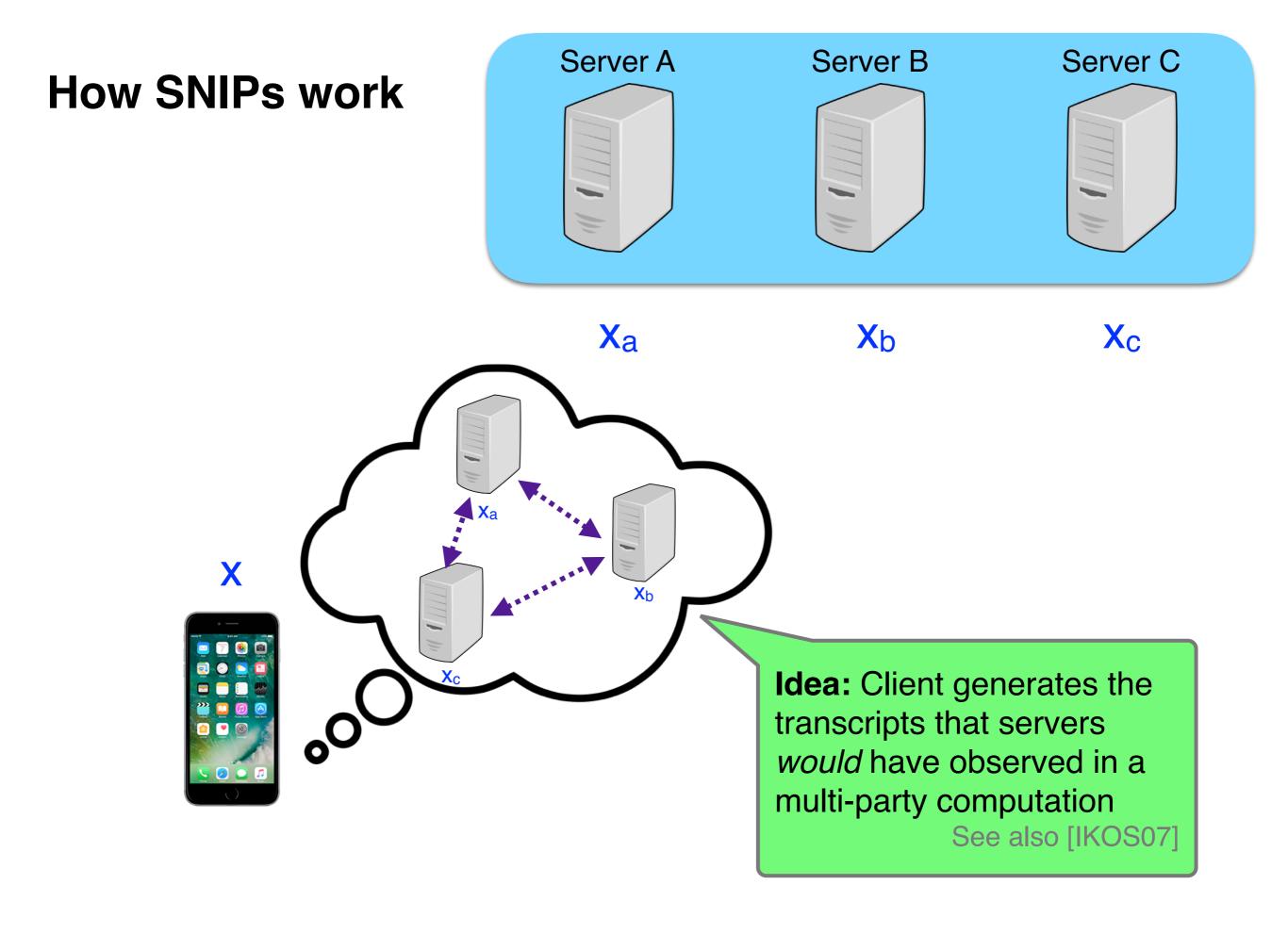


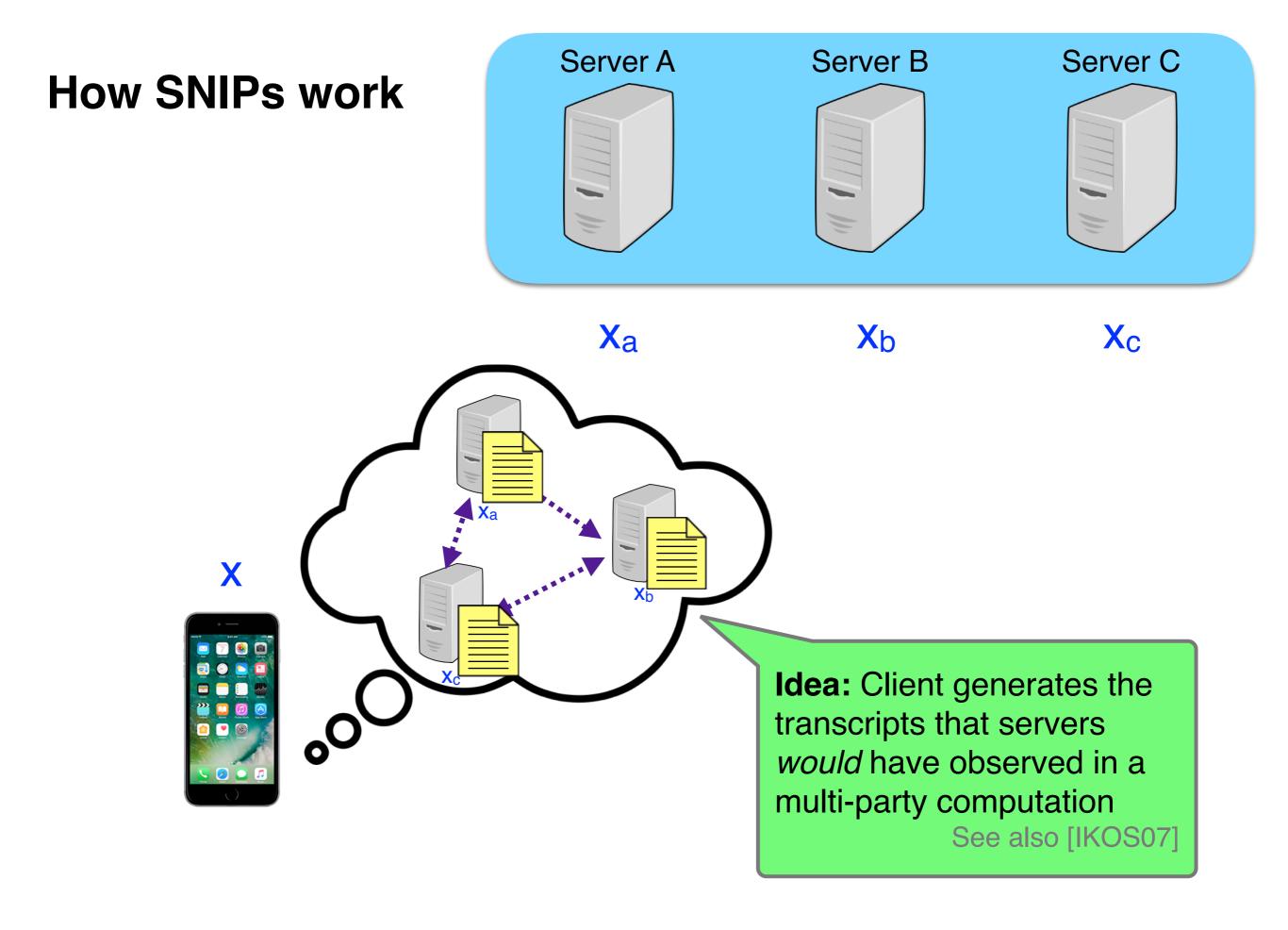


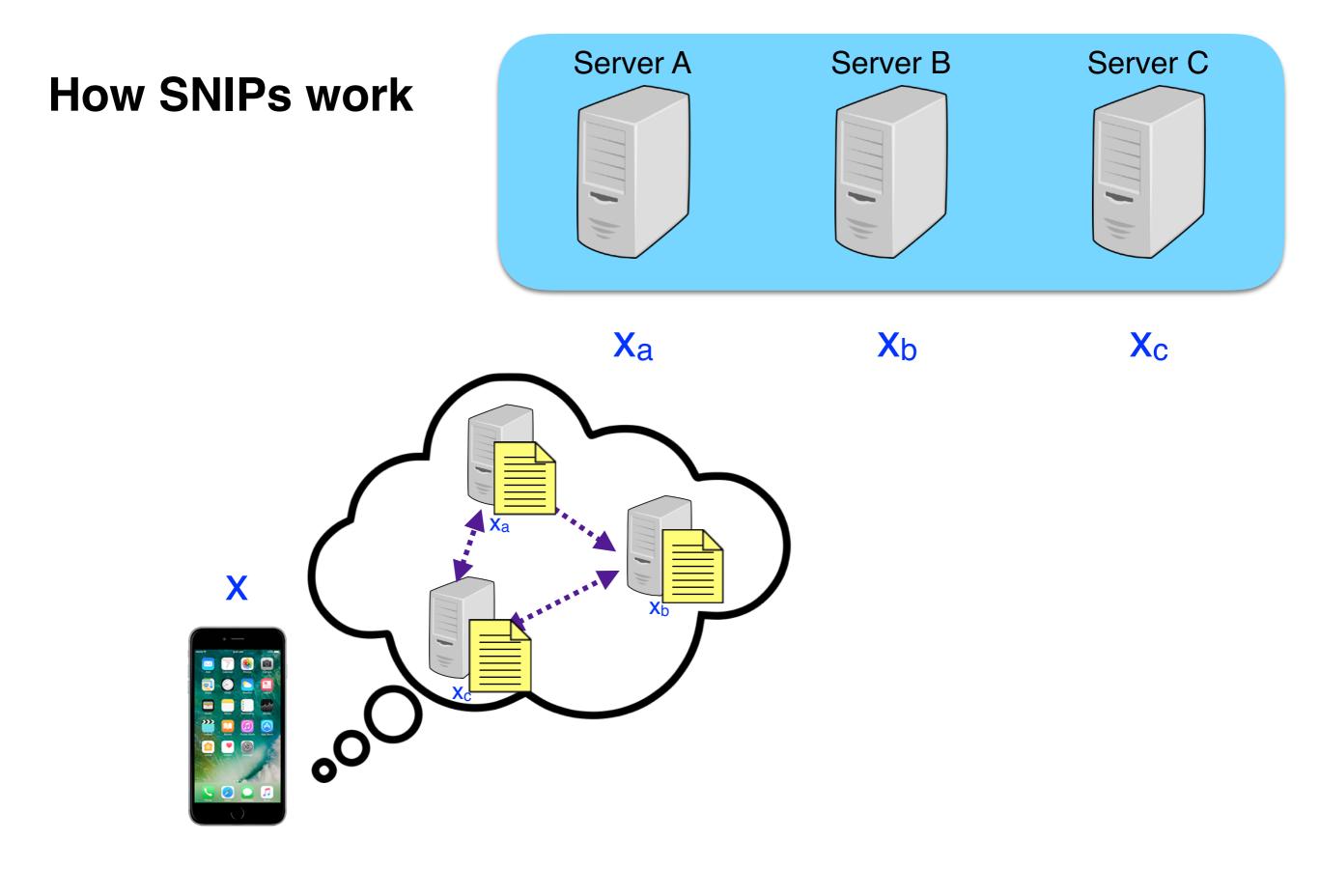
1 💌 🔞

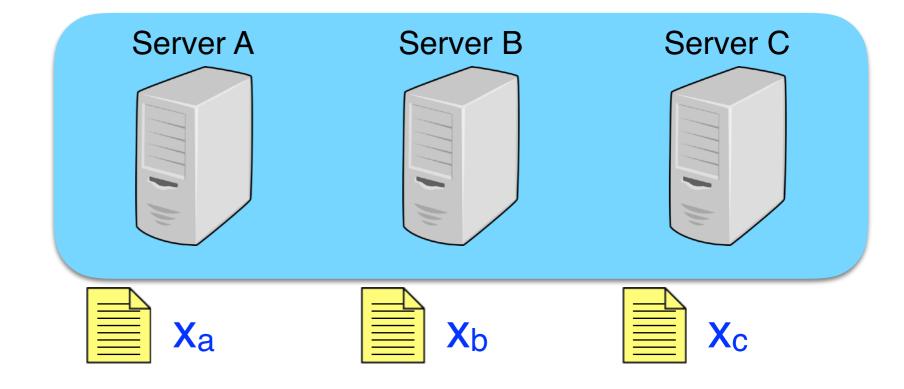
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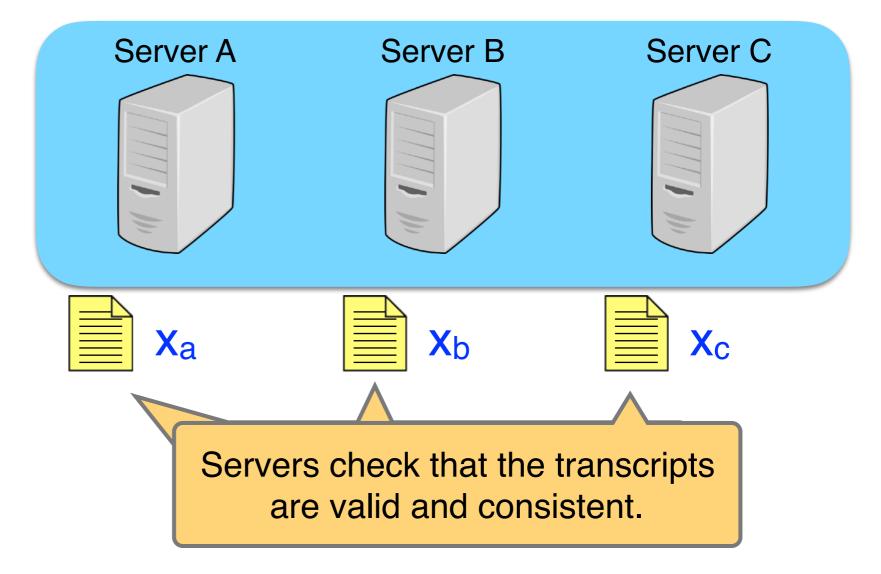






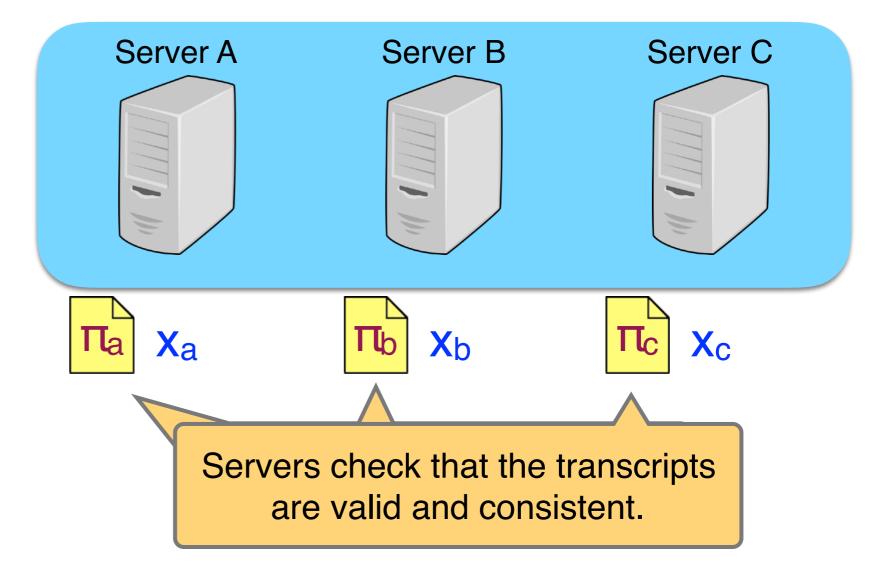






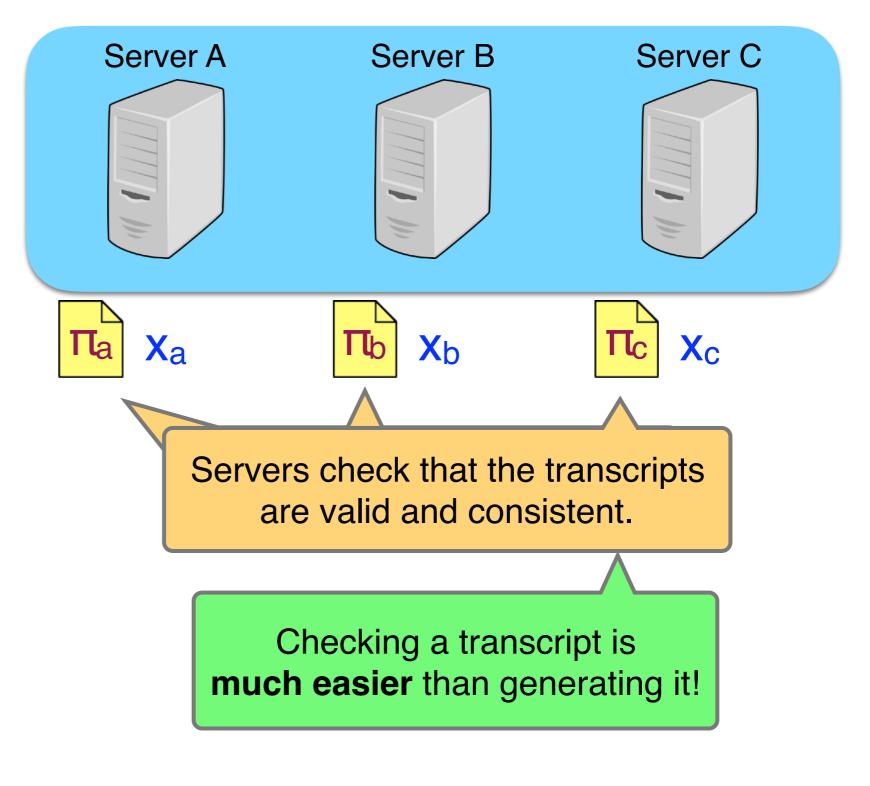
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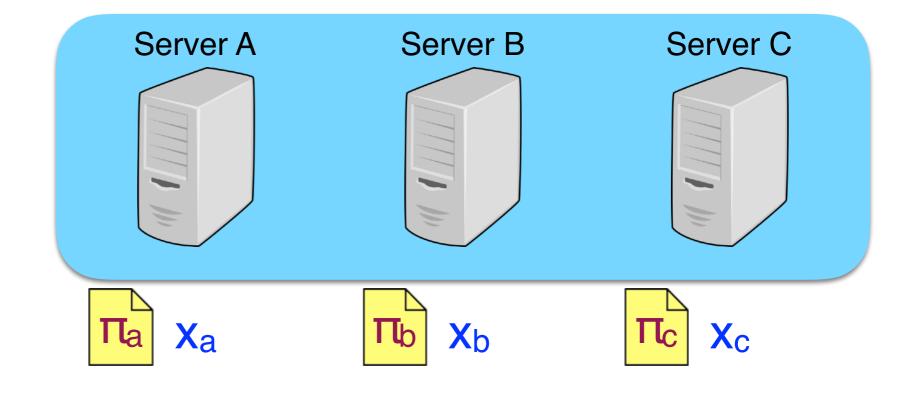
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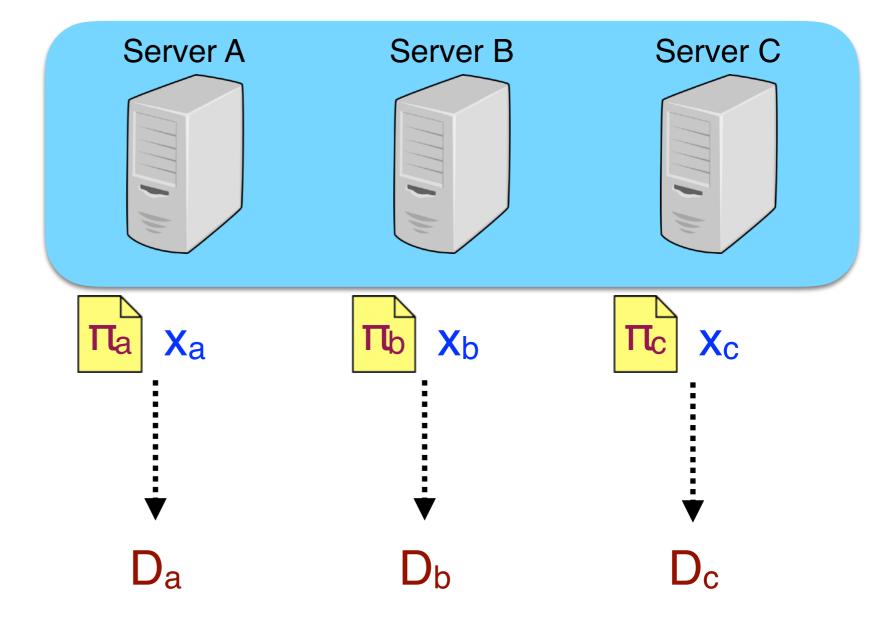


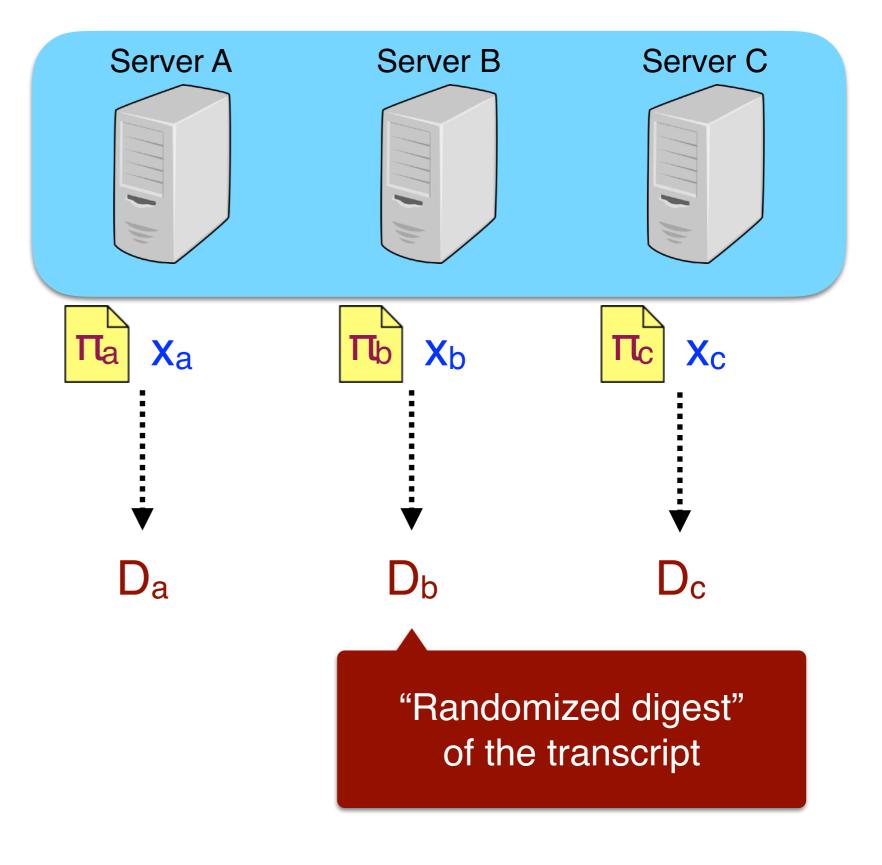


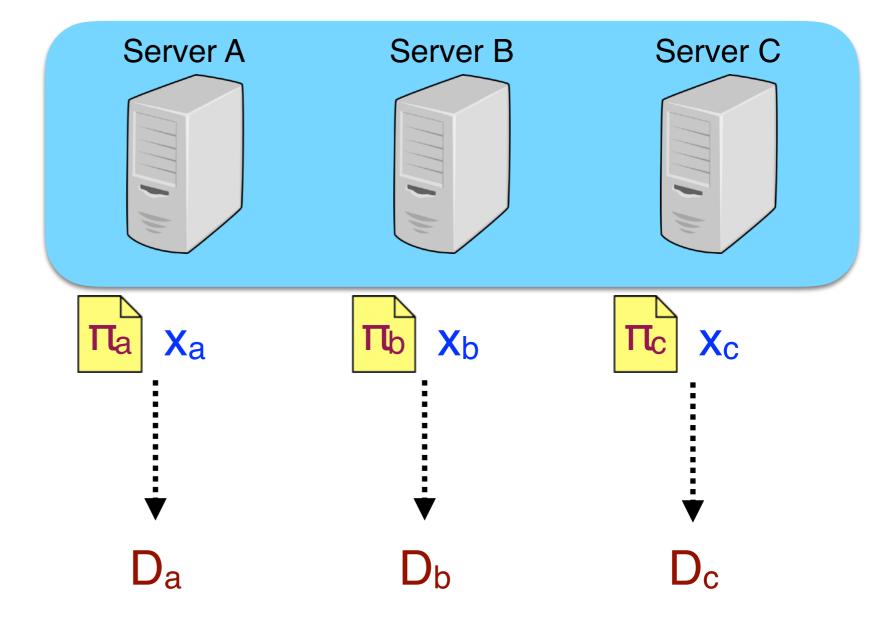
Χ

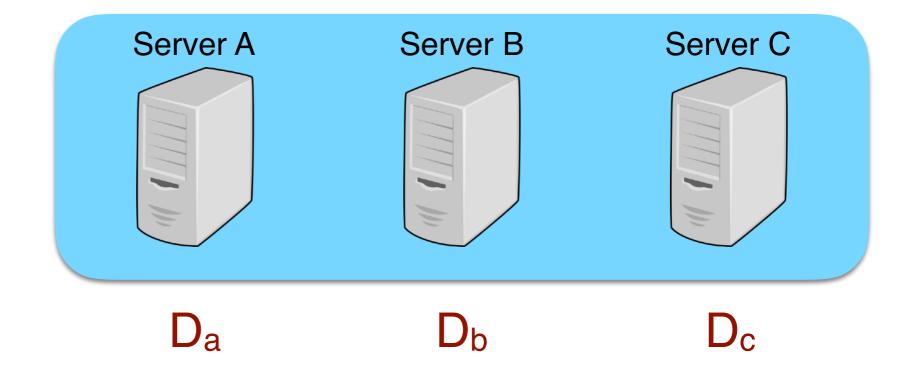


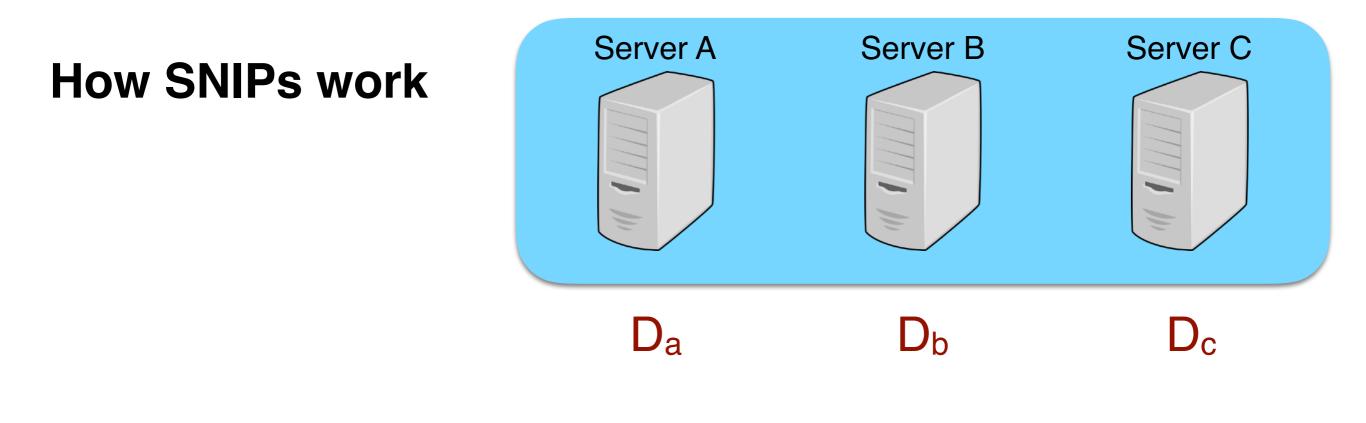




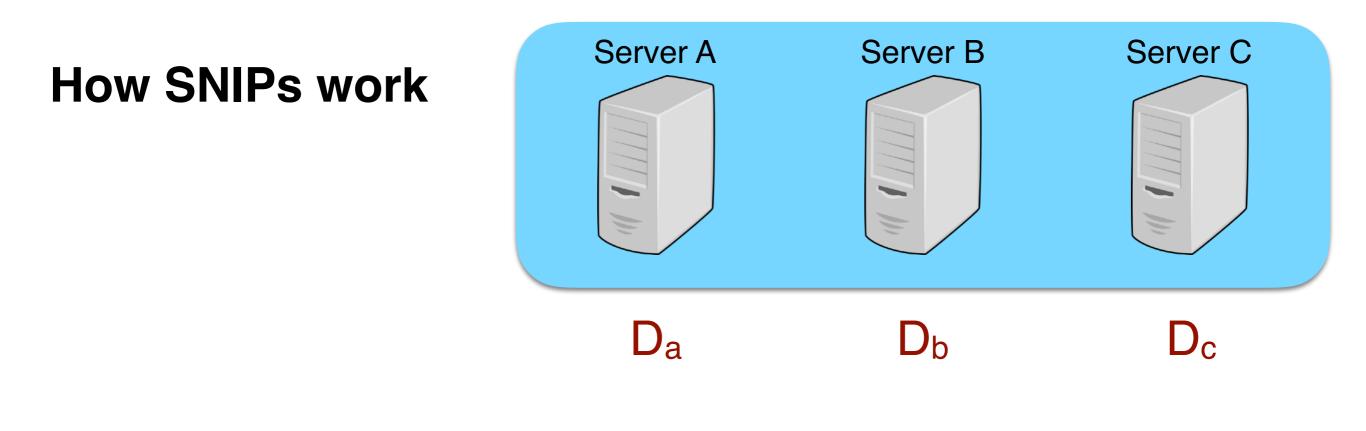








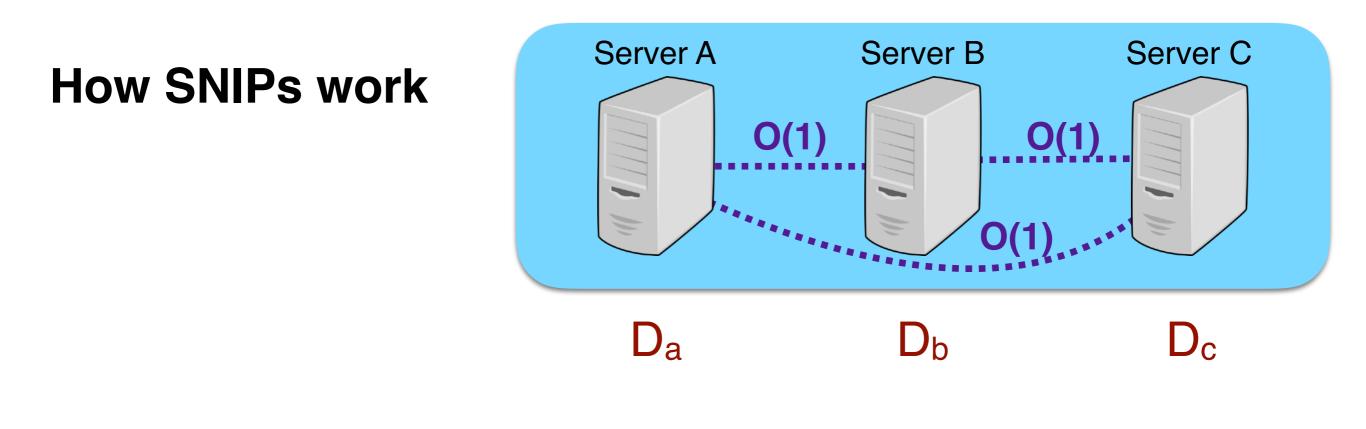




- If x is valid,  $D_a + D_b + D_c = 0$
- If x is invalid,  $D_a + D_b + D_c \neq 0$  with high probability

Servers run lightweight multi-party computation to check that  $D_a + D_b + D_c = 0$ 

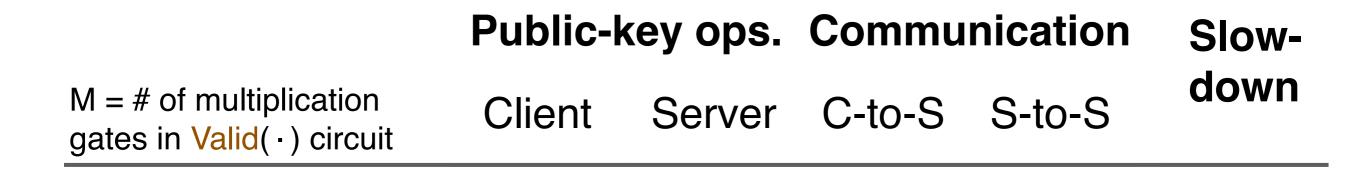
If so, servers accept x is valid.

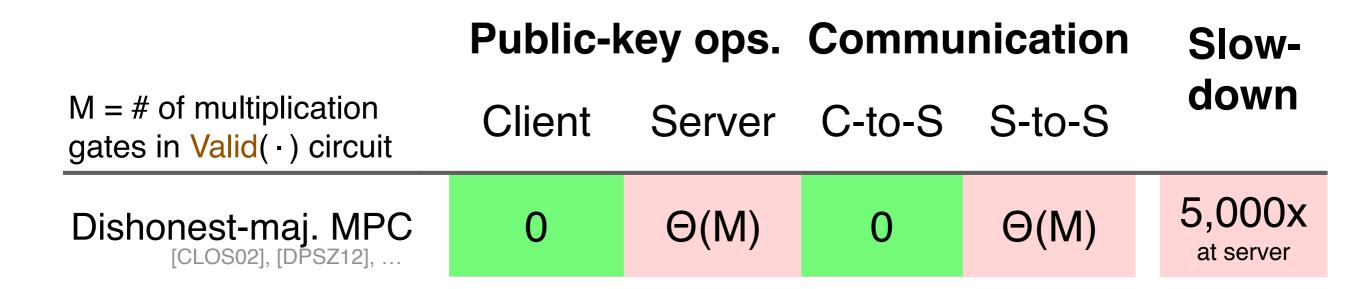


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	Public-key ops.		Communication		Slow-
$M = #$ of multiplication gates in Valid( $\cdot$ ) circuit	Client	Server	C-to-S	S-to-S	down
Dishonest-maj. MPC [CLOS02], [DPSZ12],	0	Θ(M)	0	Θ(M)	5,000x at server
Commits + NIZKs [FS86], [CP92], [CS97],	Θ(M)	Θ(M)	Θ(M)	Θ(M)	50x at server

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For specific Valid() circuits, it is possible to eliminate this cost [BGI16]

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- A straw-man solution for private sums
- Providing robustness with SNIPs
- Evaluation
- Encodings for <u>complex</u> aggregates

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- 2. No robustness ("straw man")
- 3. Prio (privacy + robustness)
- 4. NIZK (privacy + robustness)



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E.g., for privately measuring telemetry data.

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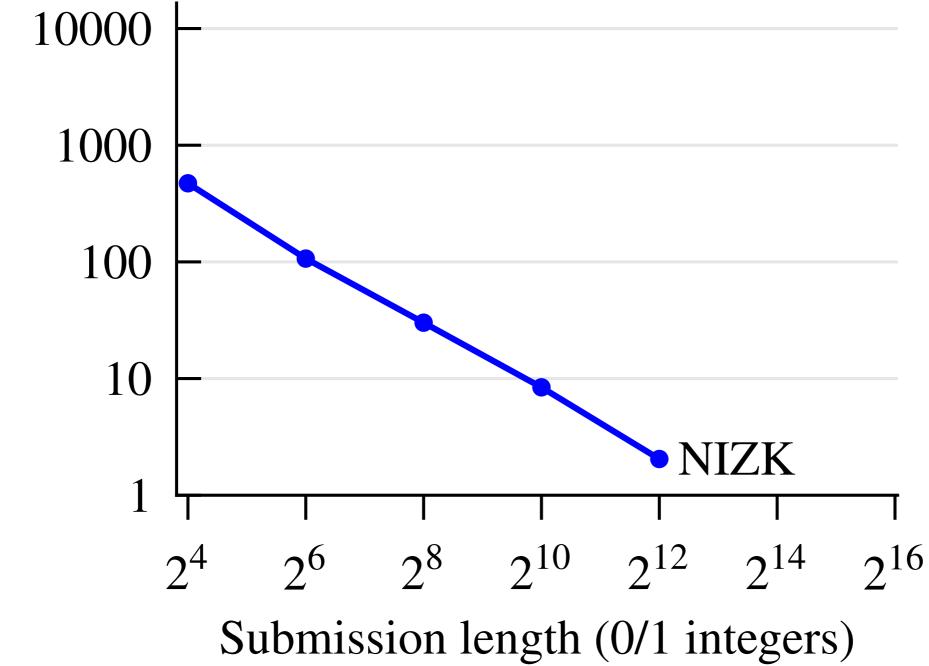


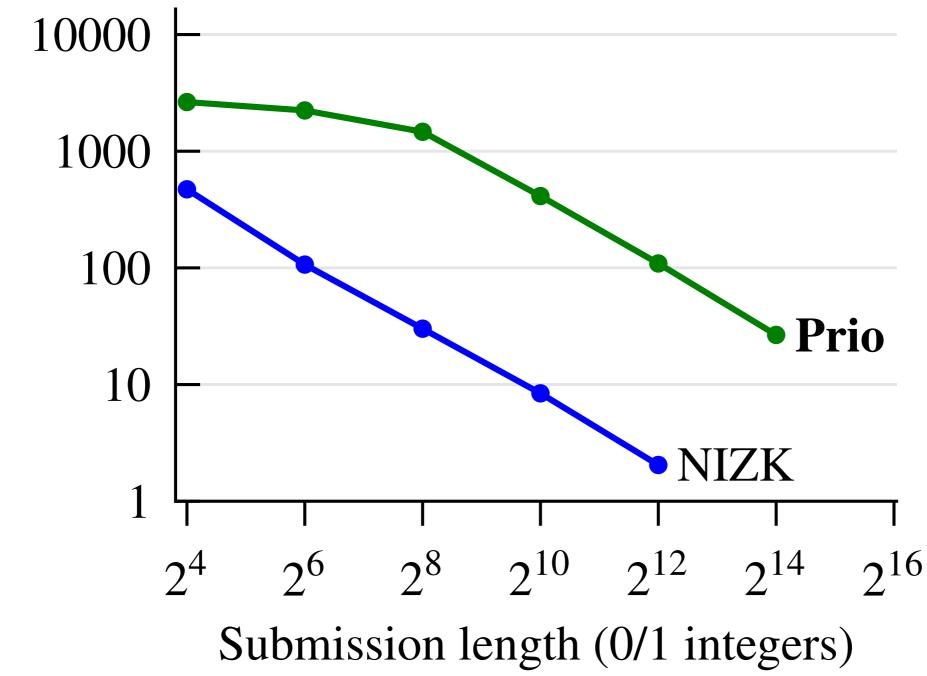
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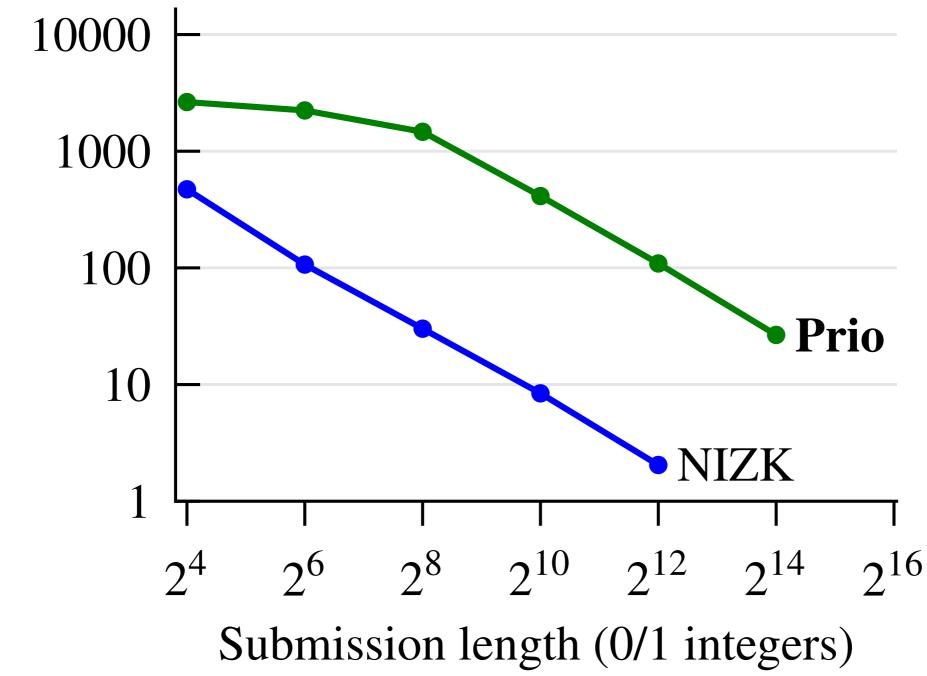
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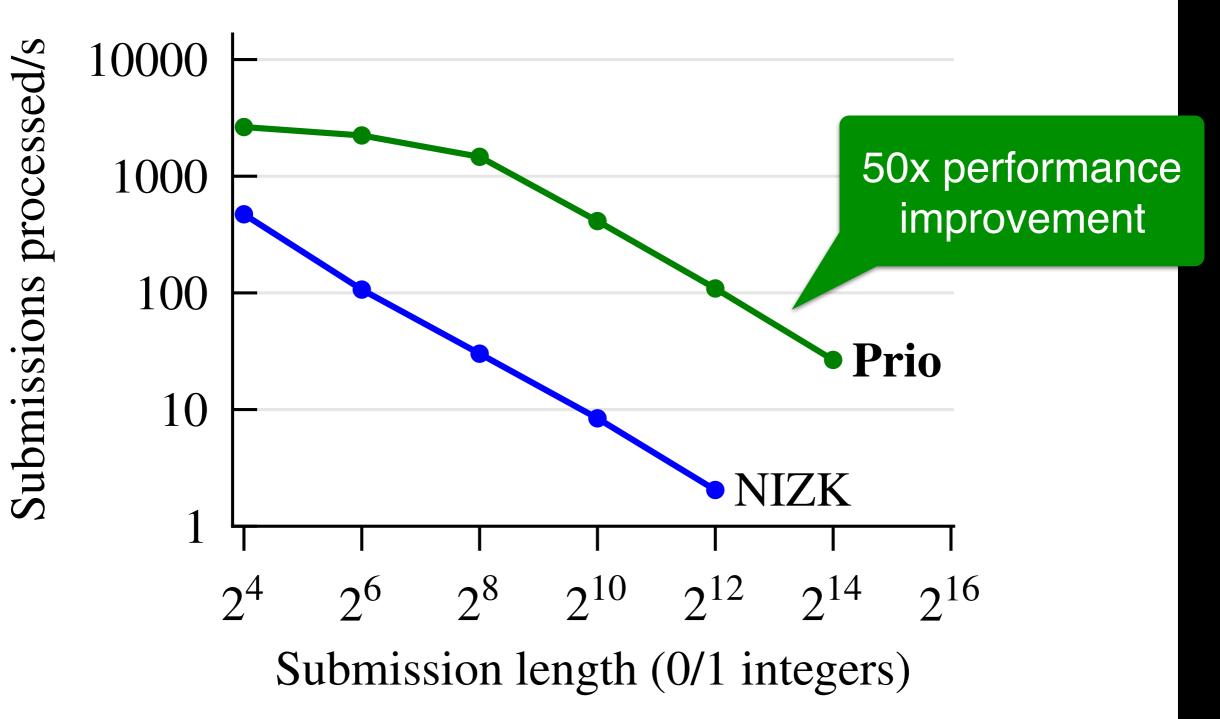
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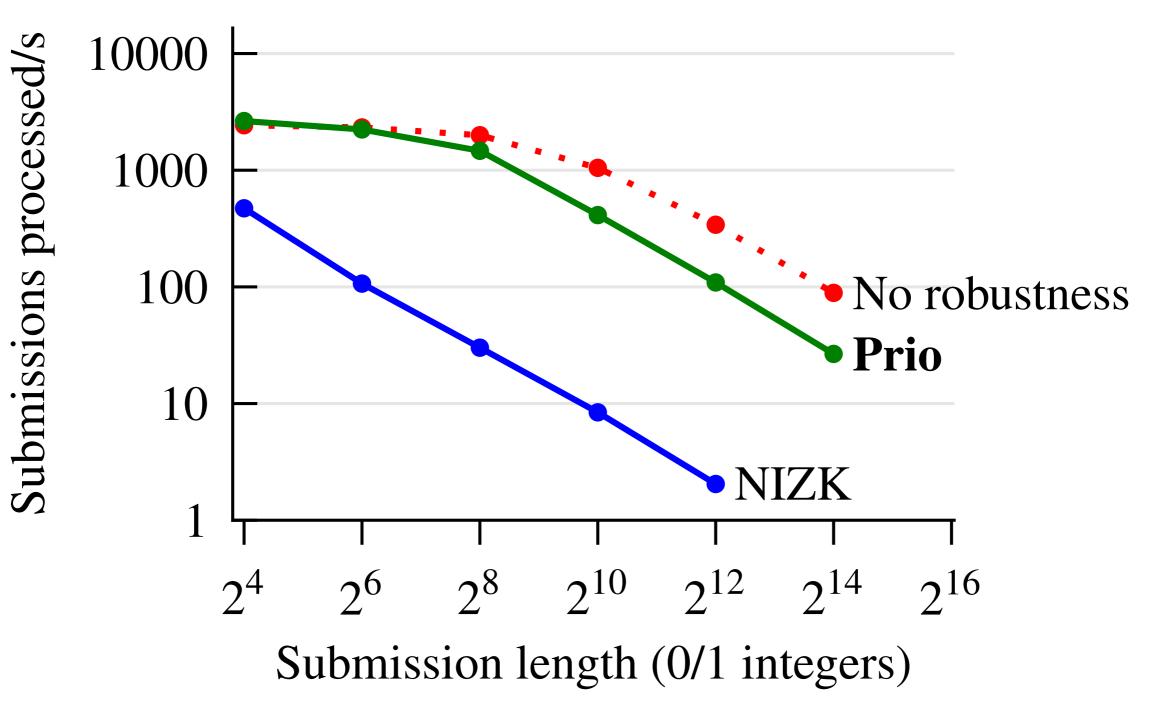


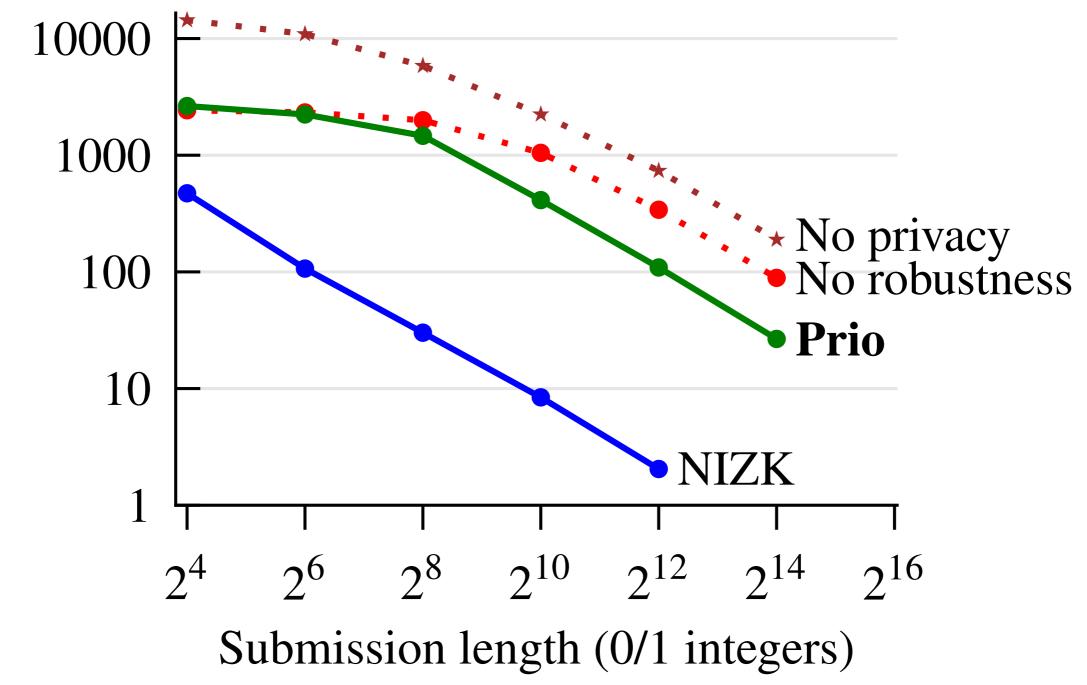


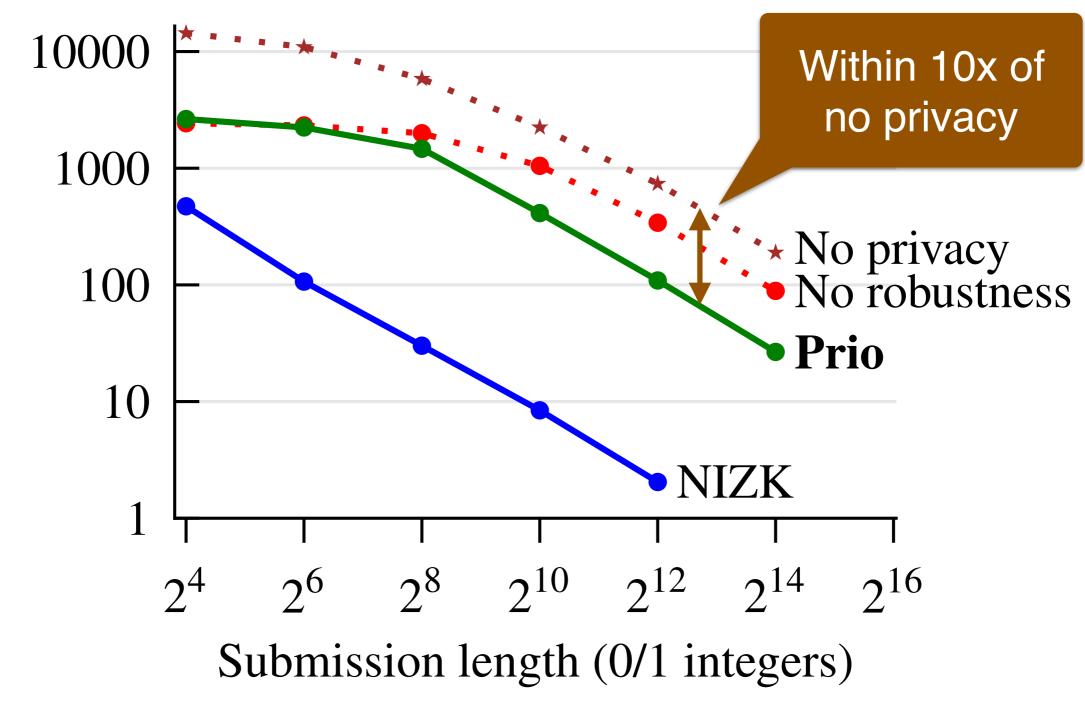


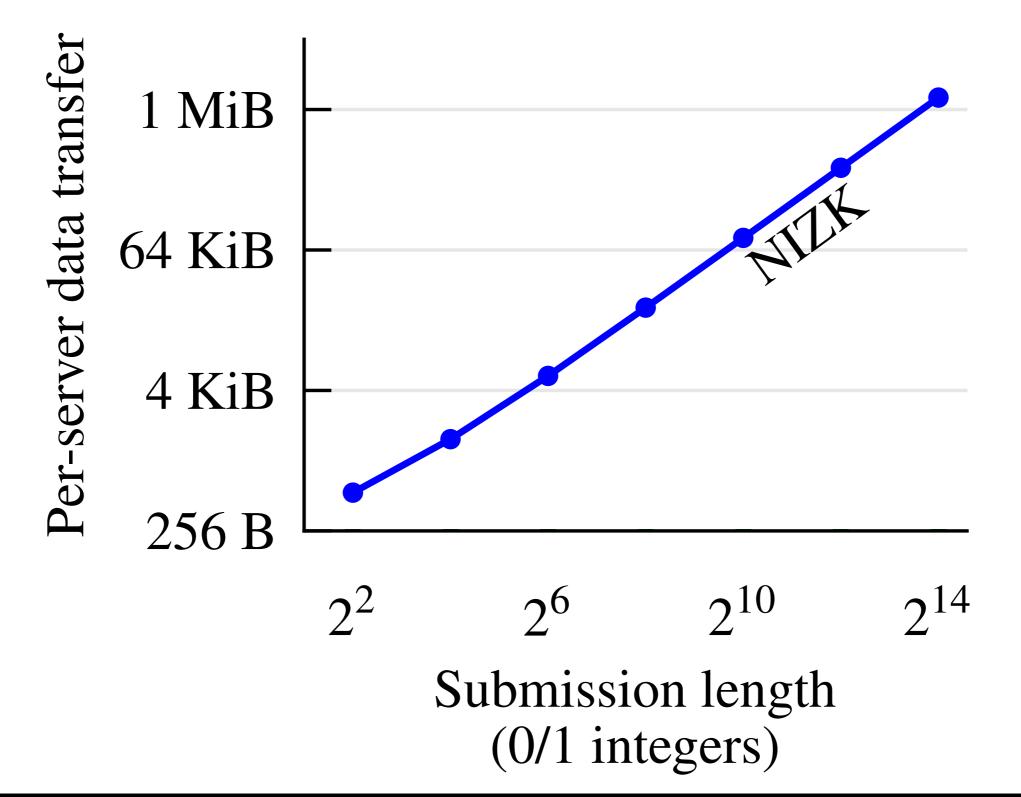


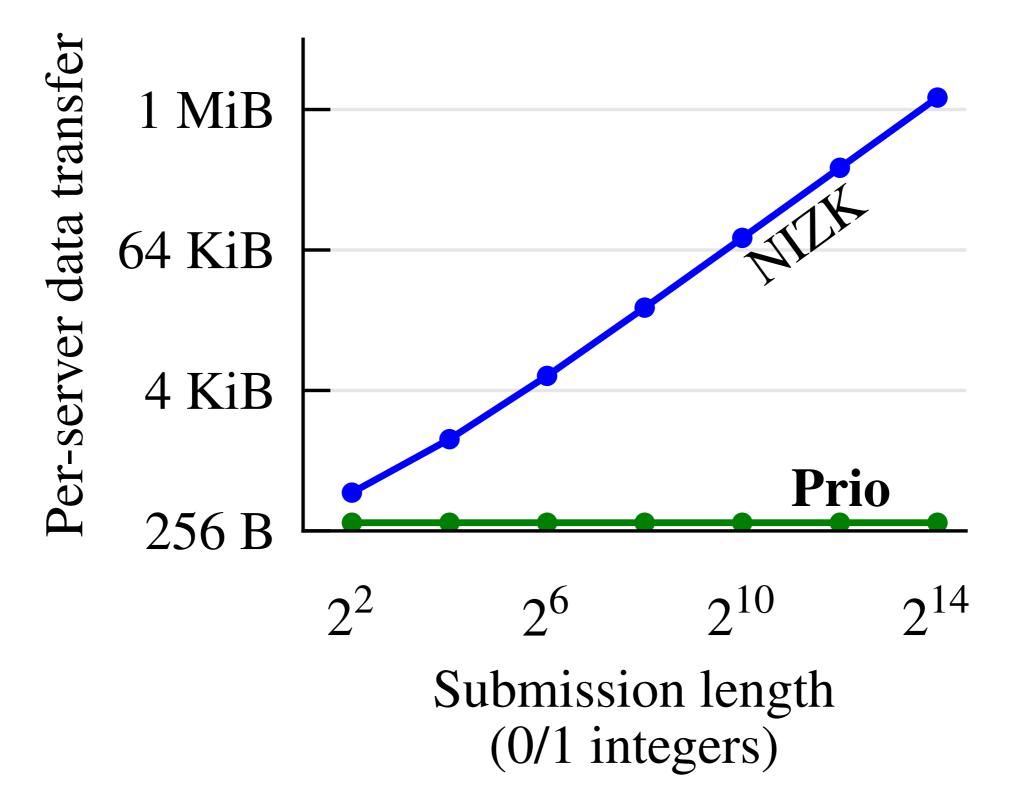


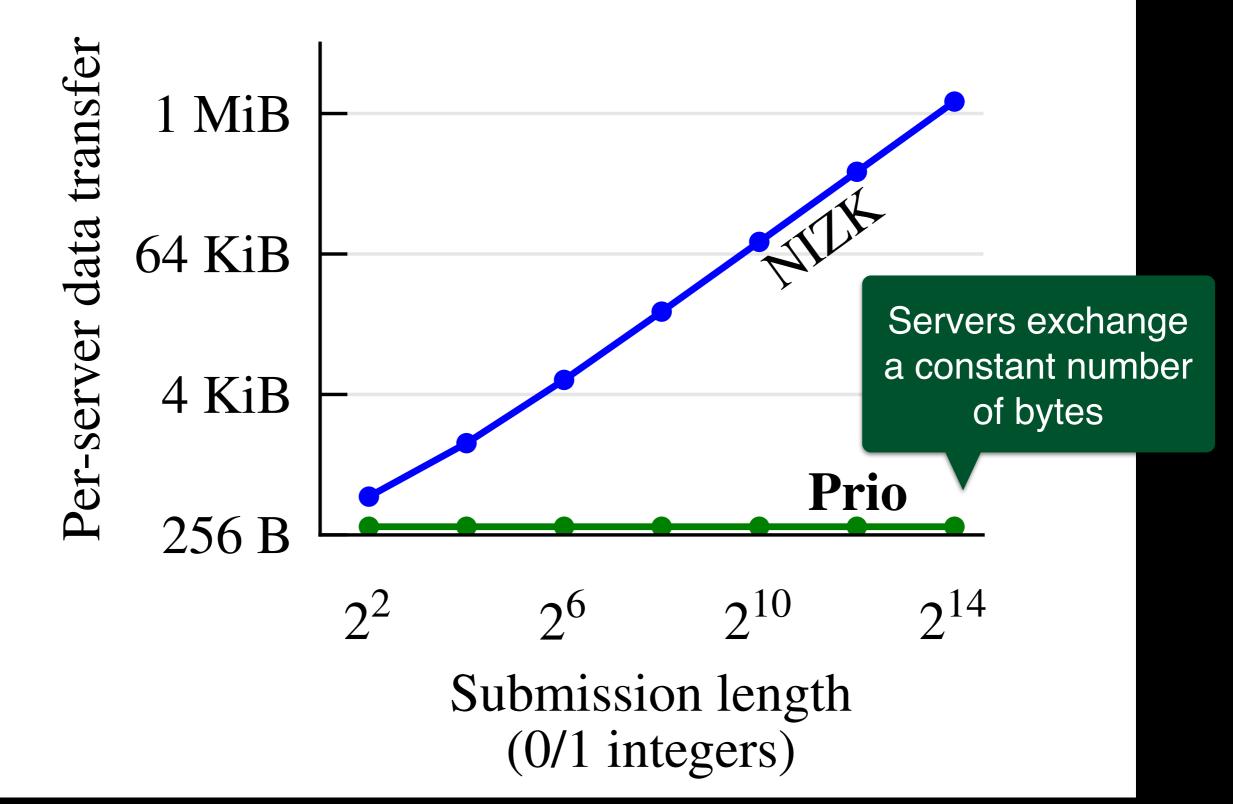












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If you can compute private sums, you can compute many other interesting aggregates using known techniques

[PrivStats11], [KDK11], [DFKZ13], [PrivEx14], [MDD16], ...

- Average
- Variance
- Standard deviation
- Most popular (approx)
- "Heavy hitters" (approx)
- Min and max (approx)
- Quality of arbitrary regression model (R<sup>2</sup>)
- Least-squares regression
- Stochastic gradient descent [Bonawitz et al. 2016]

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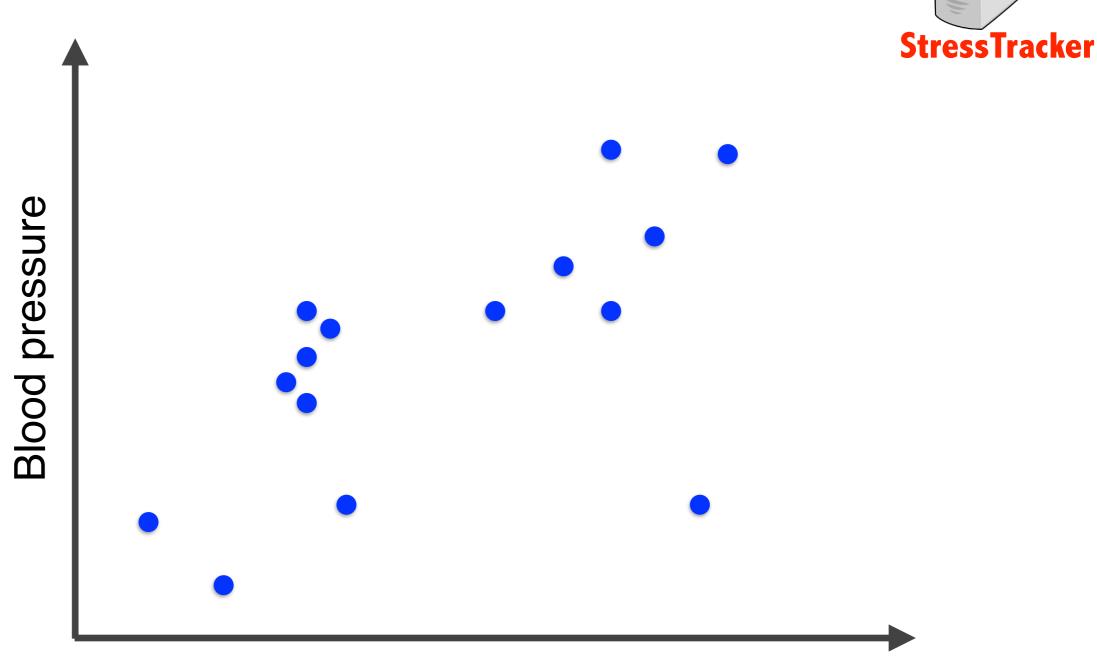
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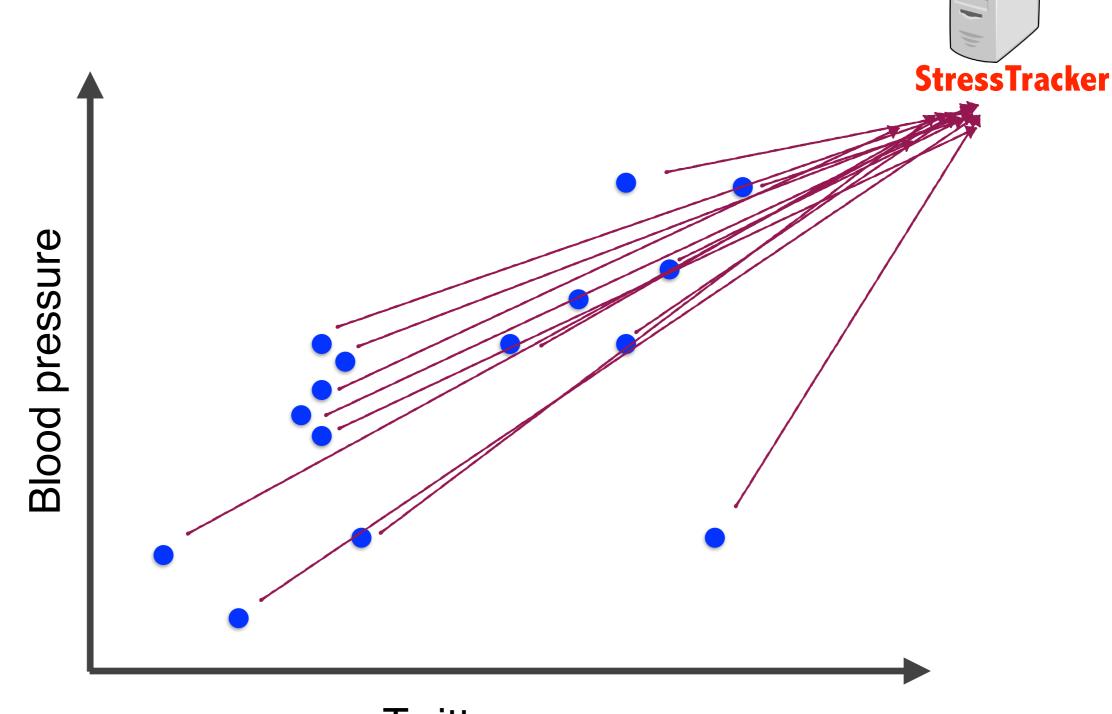
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See the paper for the details

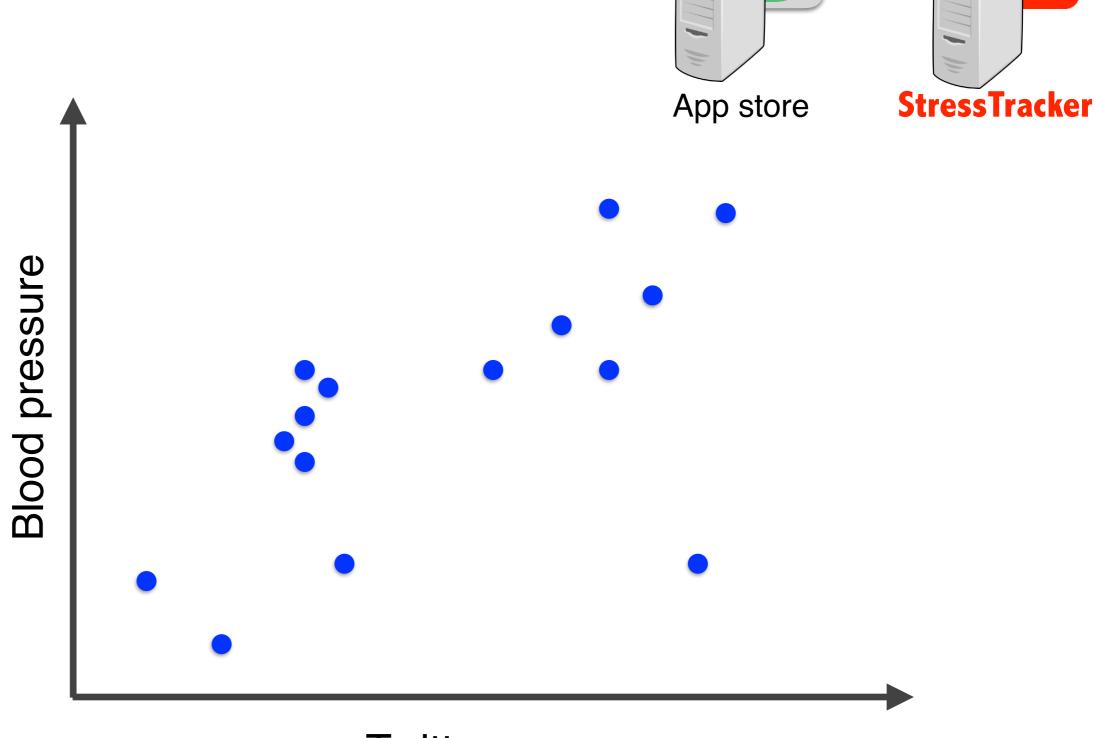
Today



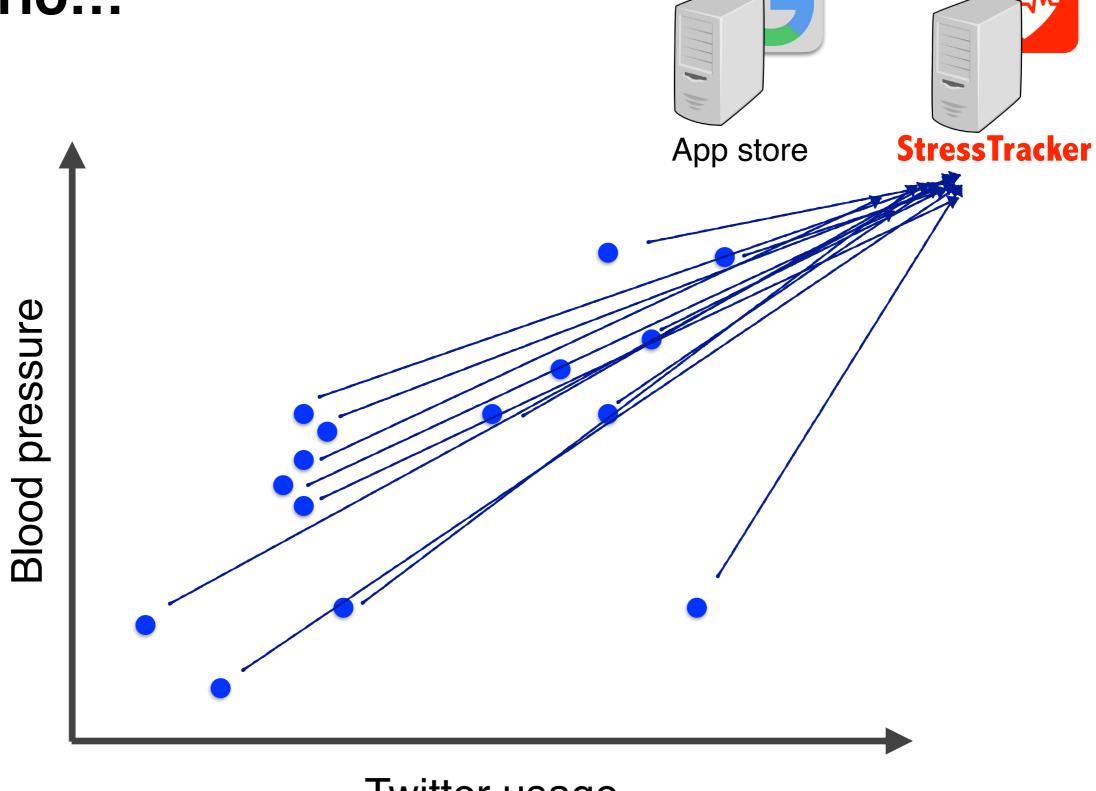
## Today



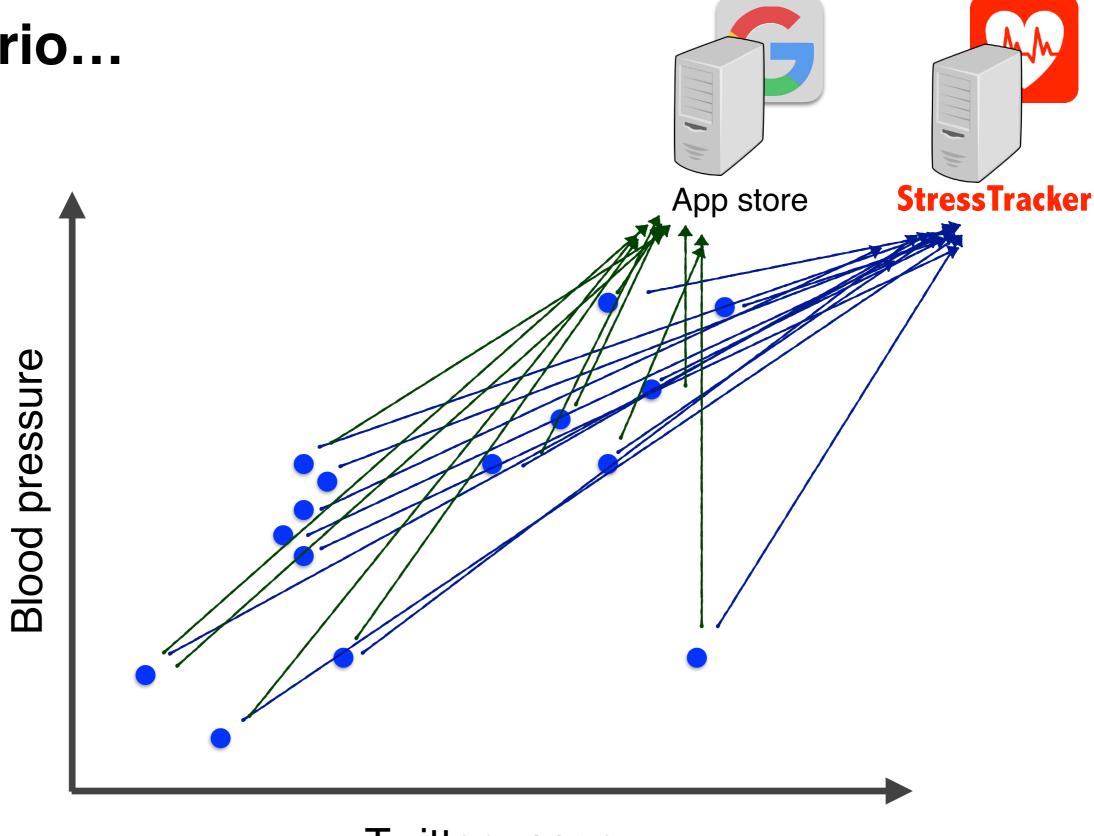
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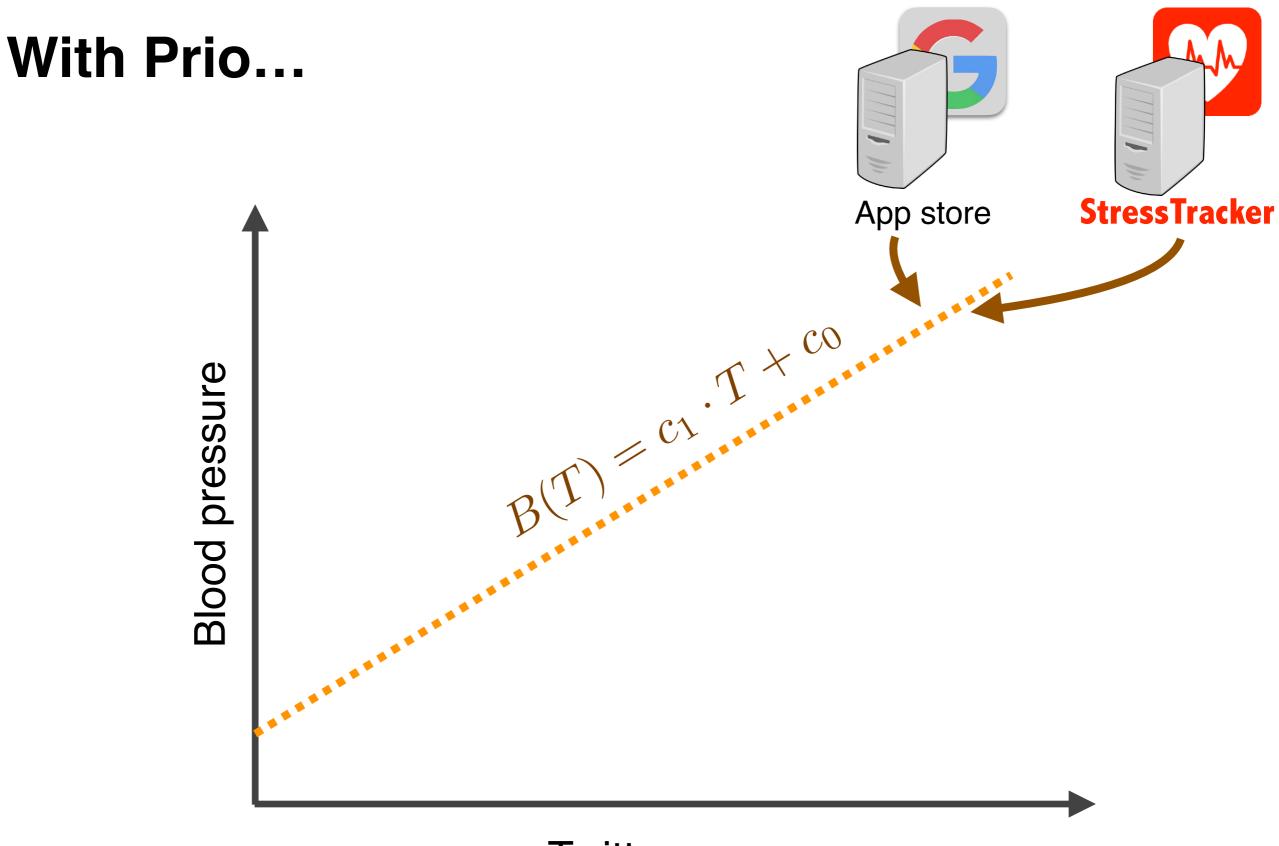


## With Prio...



## With Prio...





# Conclusions

- Wholesale collection of sensitive user data puts our security at risk.
- Prio is the first system for aggregation that provides:
  - exact correctness,
  - privacy,
  - robustness, and
  - efficiency.
- To do so, Prio uses SNIPs and aggregatable encodings.
- These techniques together bring private aggregation closer to practical.

## Thank you!

Henry Corrigan-Gibbs henrycg@cs.stanford.edu

https://crypto.stanford.edu/prio/

- Each of N clients holds a value Xi
- Servers want the AVG and VAR of the xis.

Each client i encodes her value x as the pair  $(x, y) = (x, x^2)$ 

Simple to check that the encoding is valid:  $Valid(x, y) = (x^2 - y)$  [outputs zero if valid]

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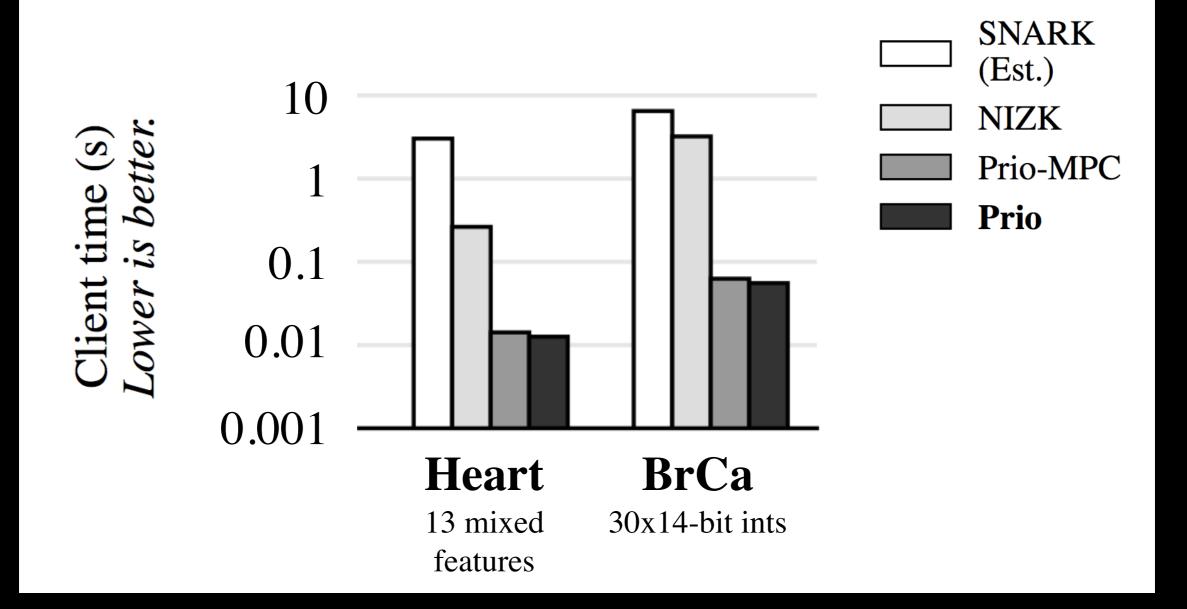
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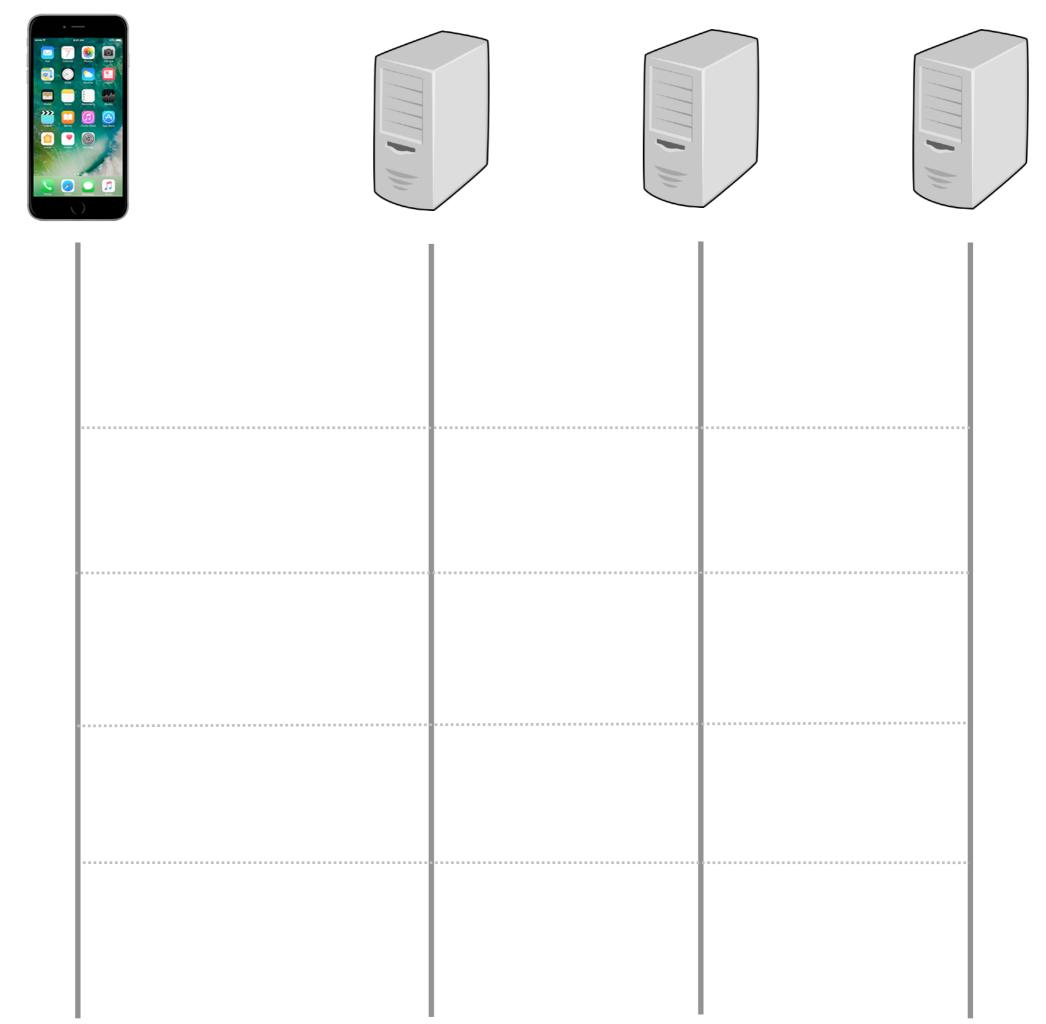
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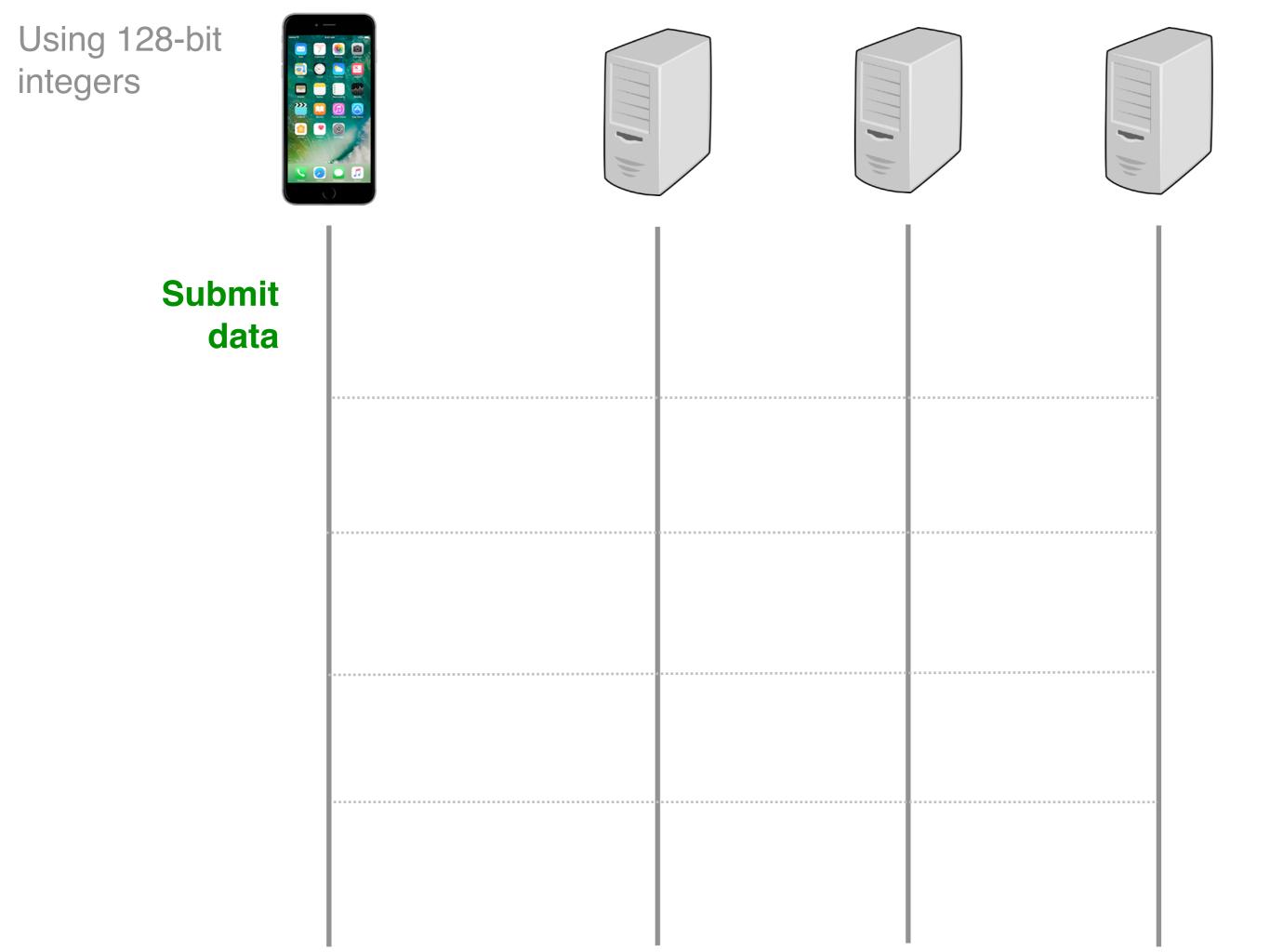
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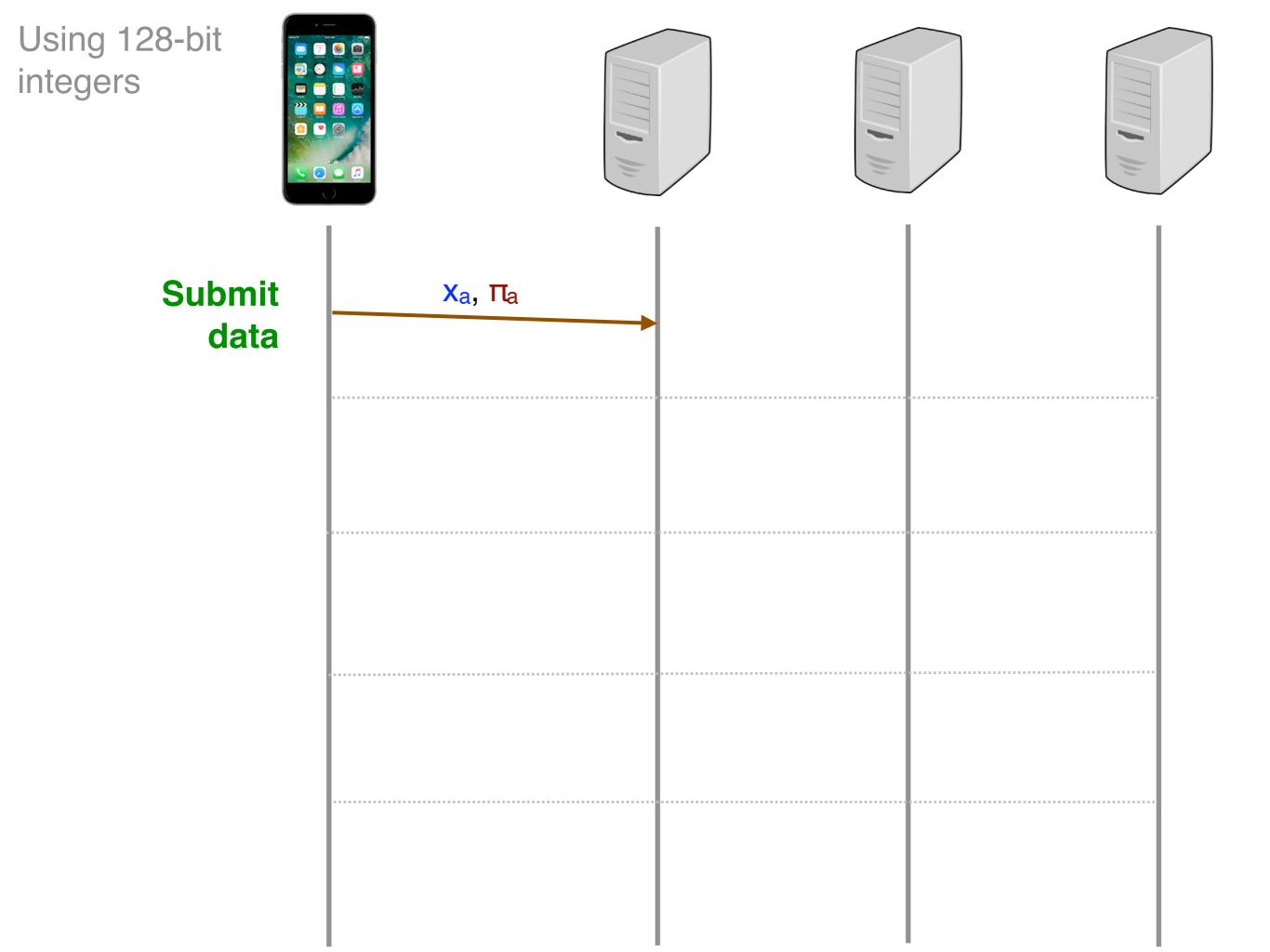
Then recover the statistics:

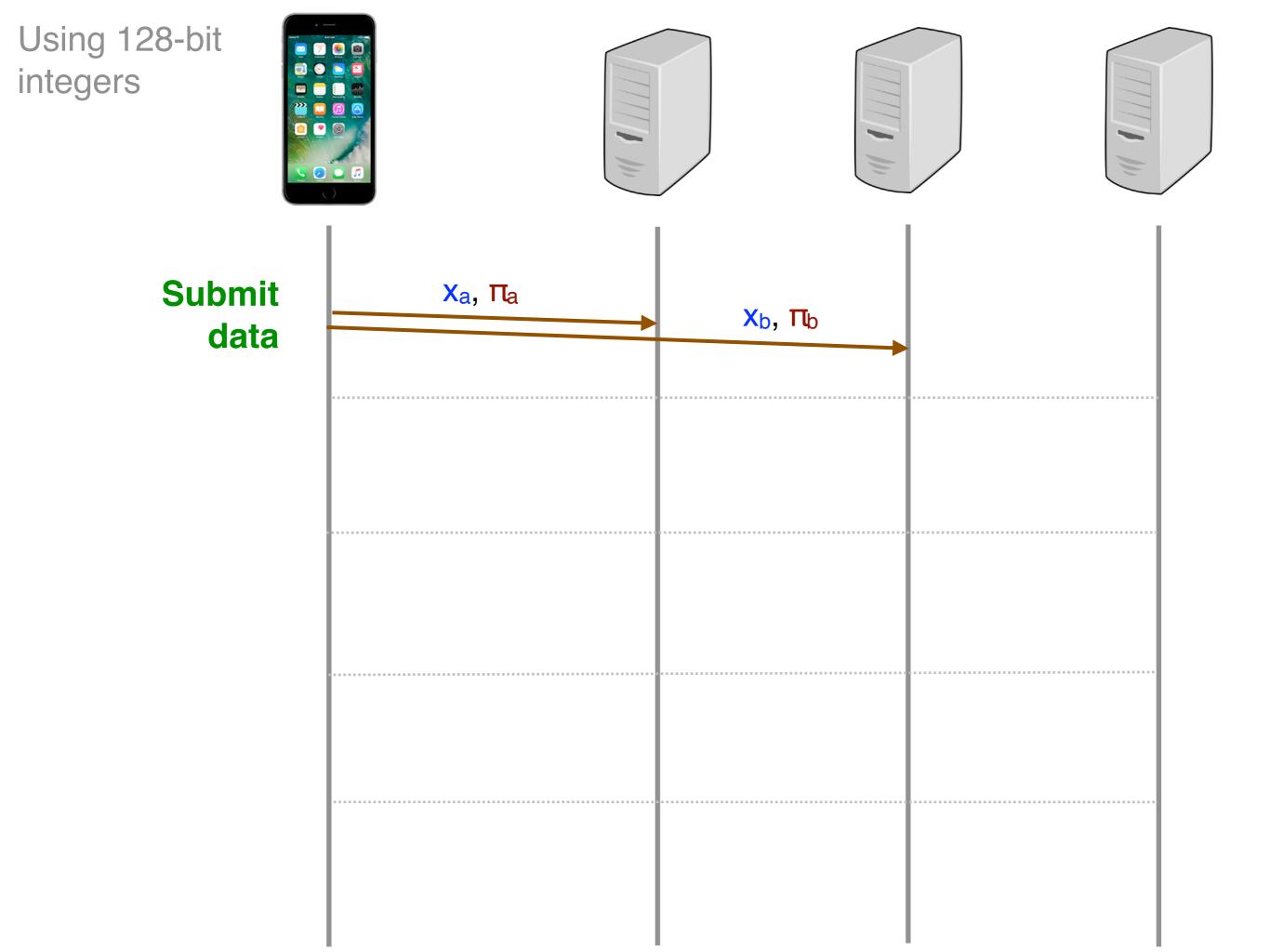


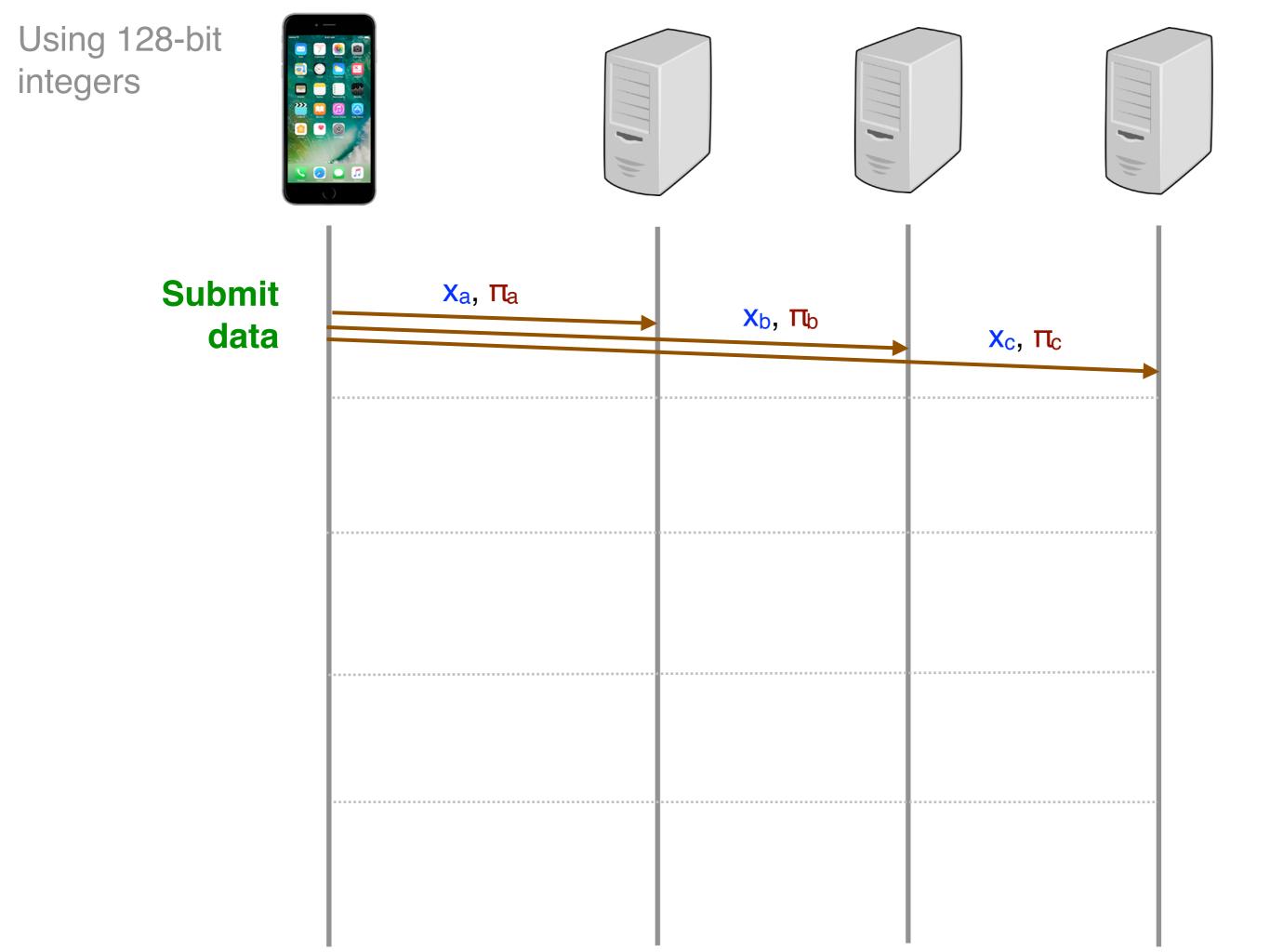
# Using 128-bit integers

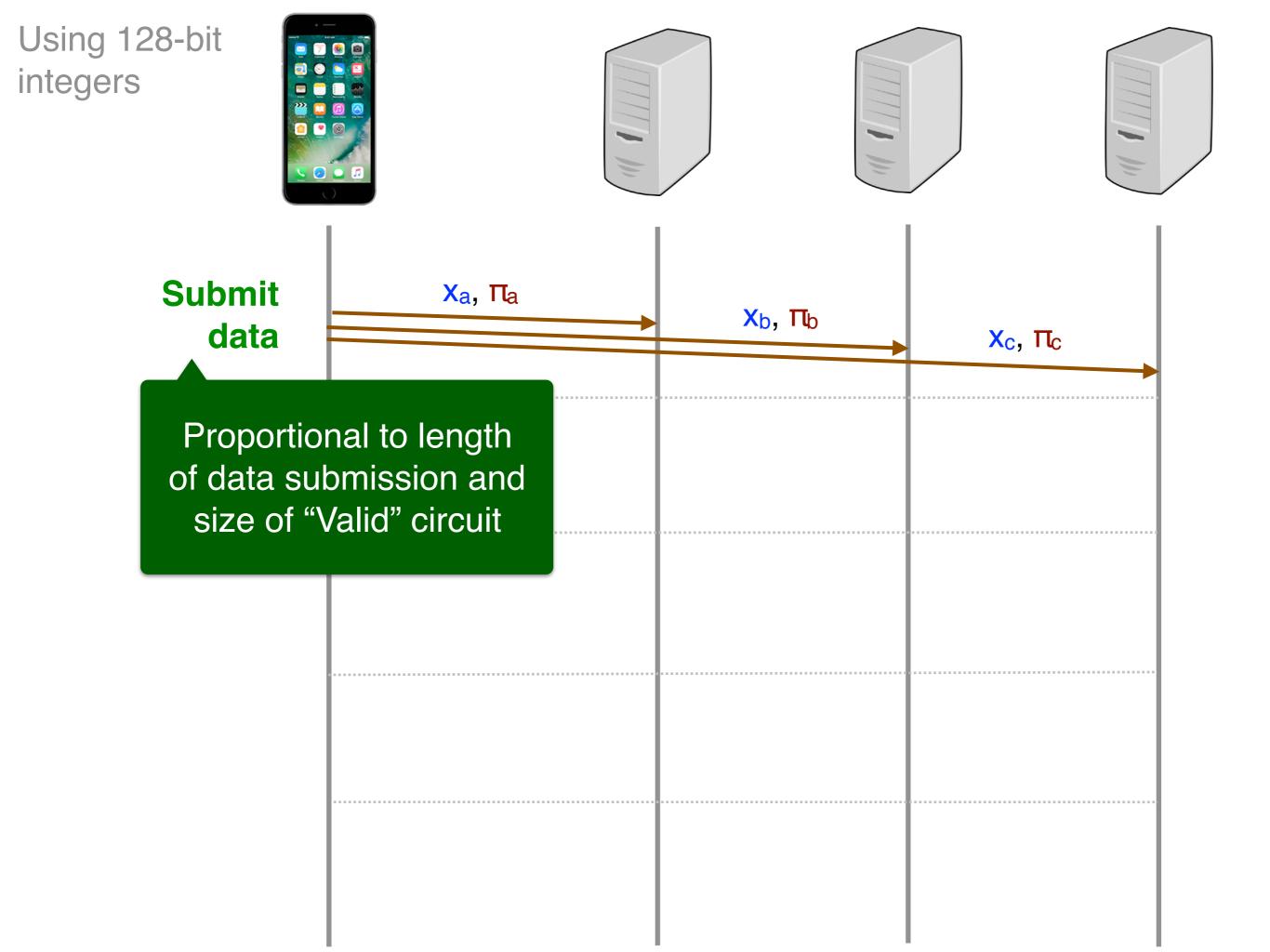


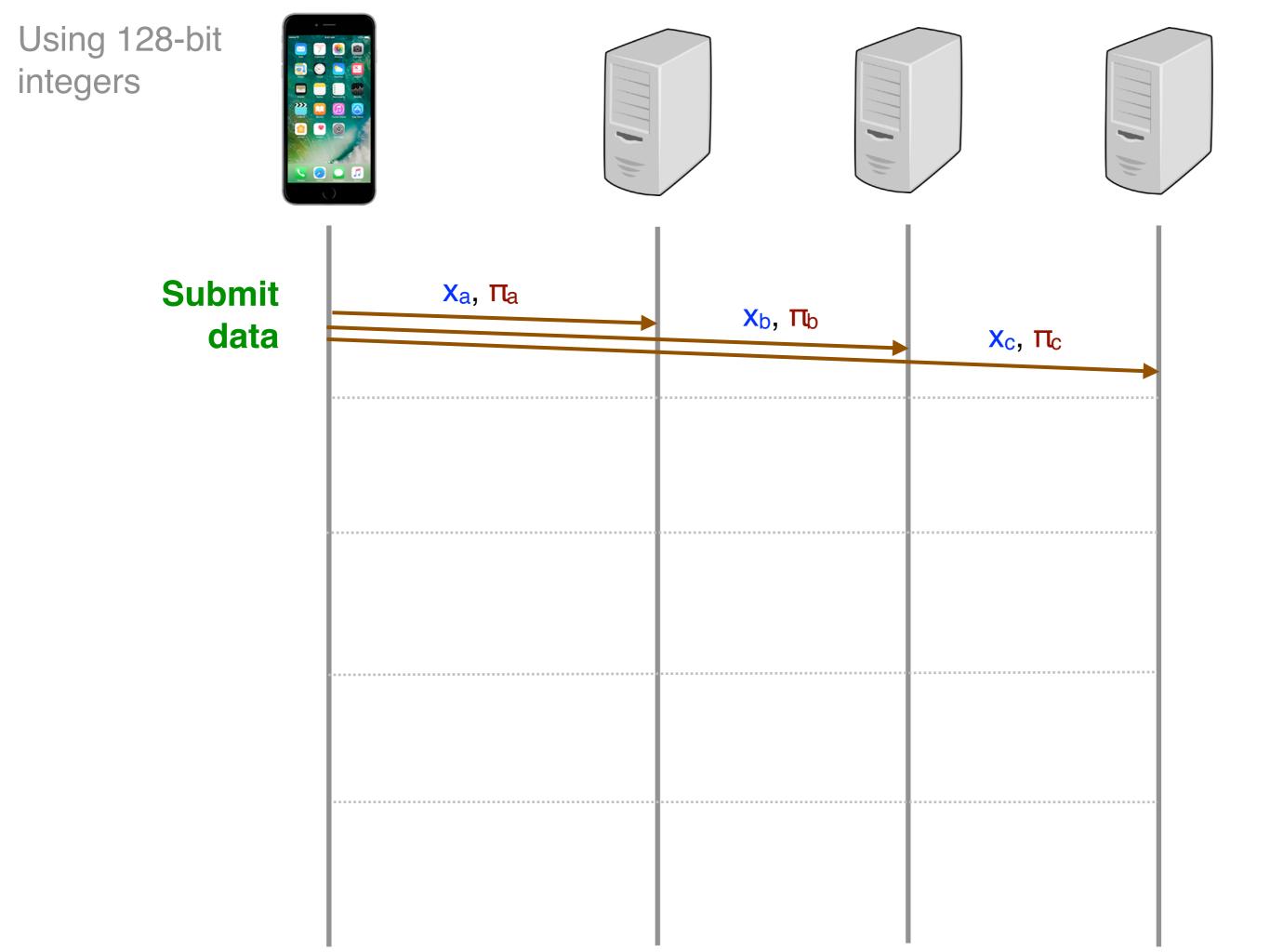


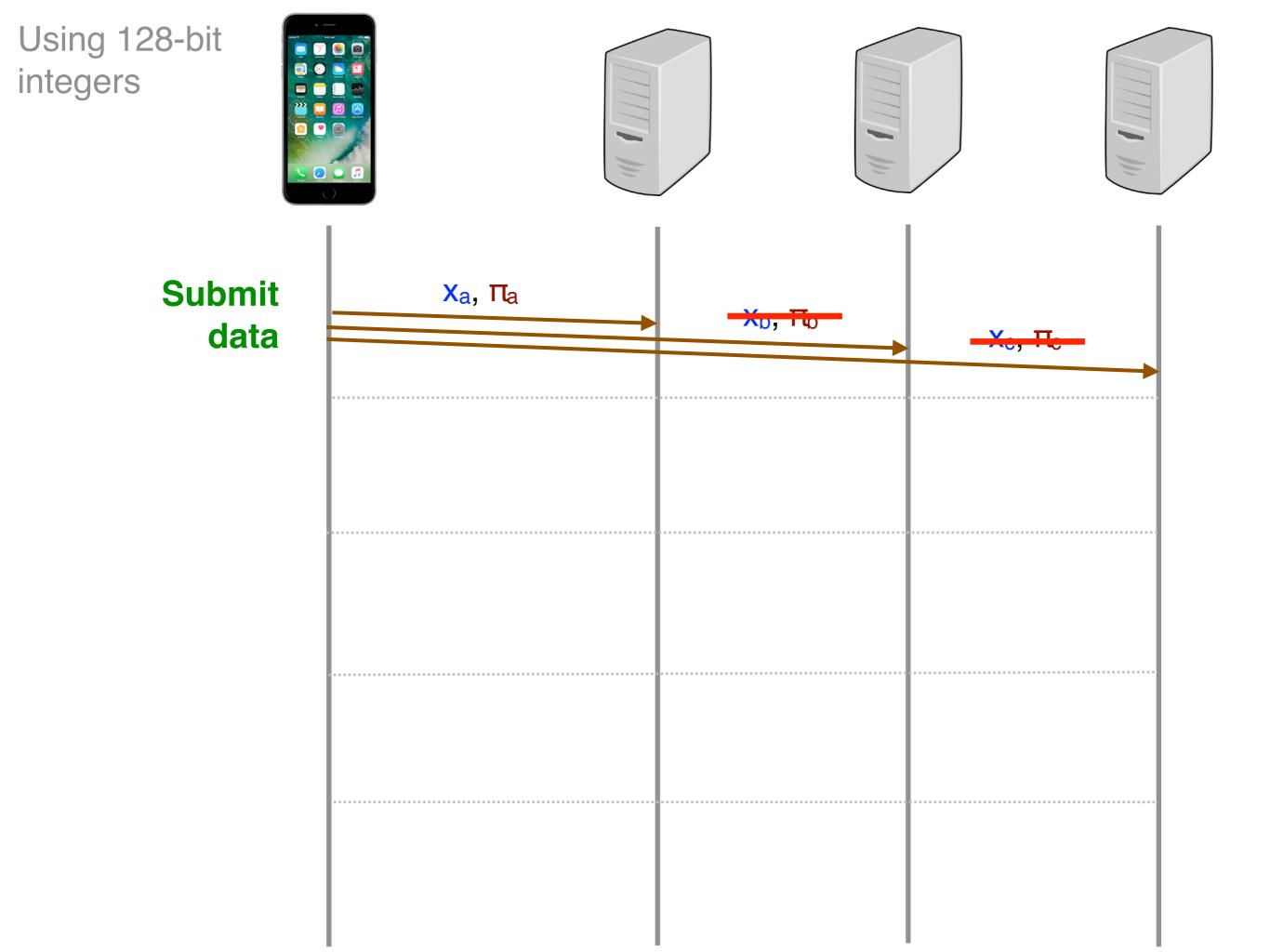


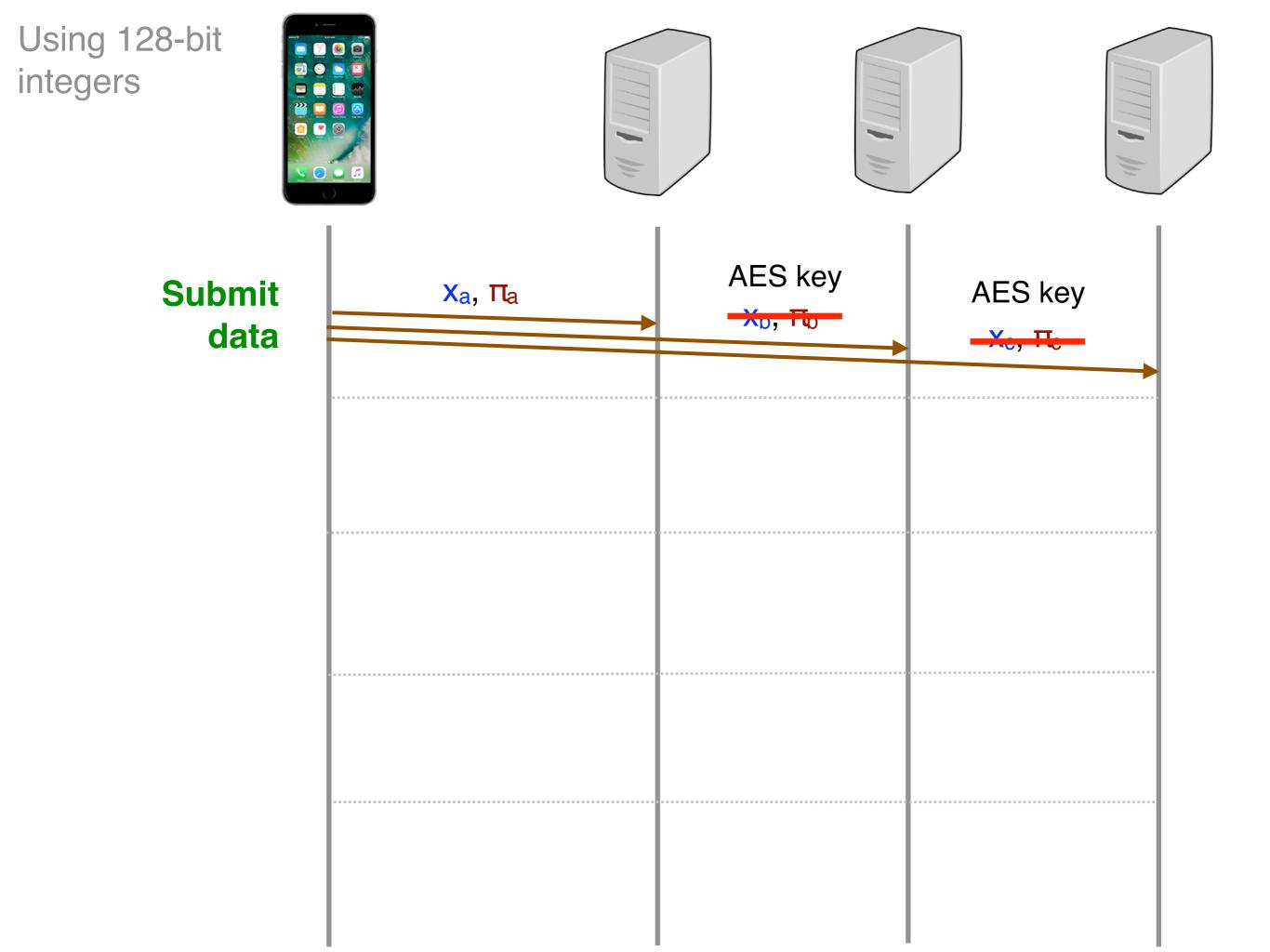


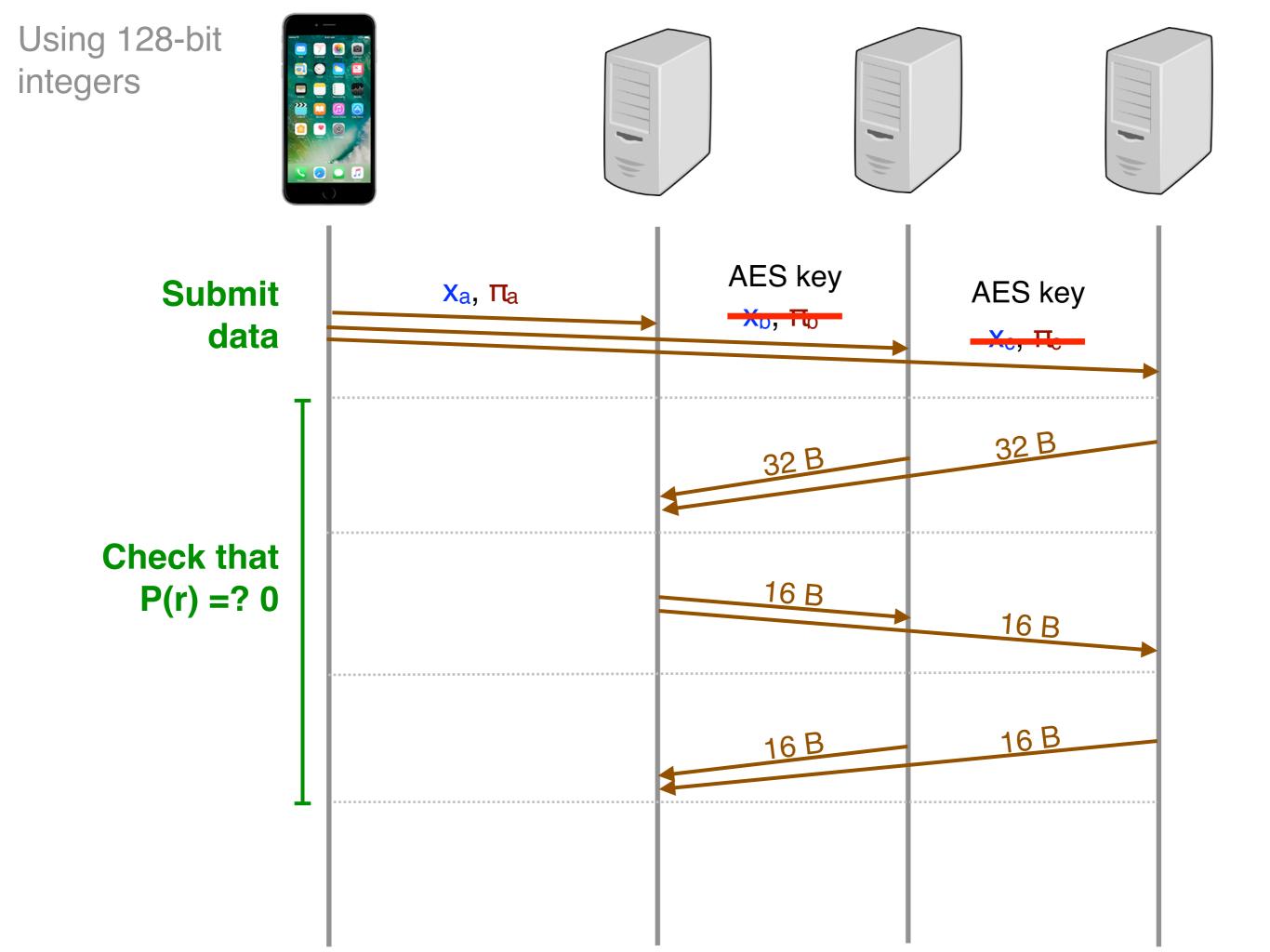


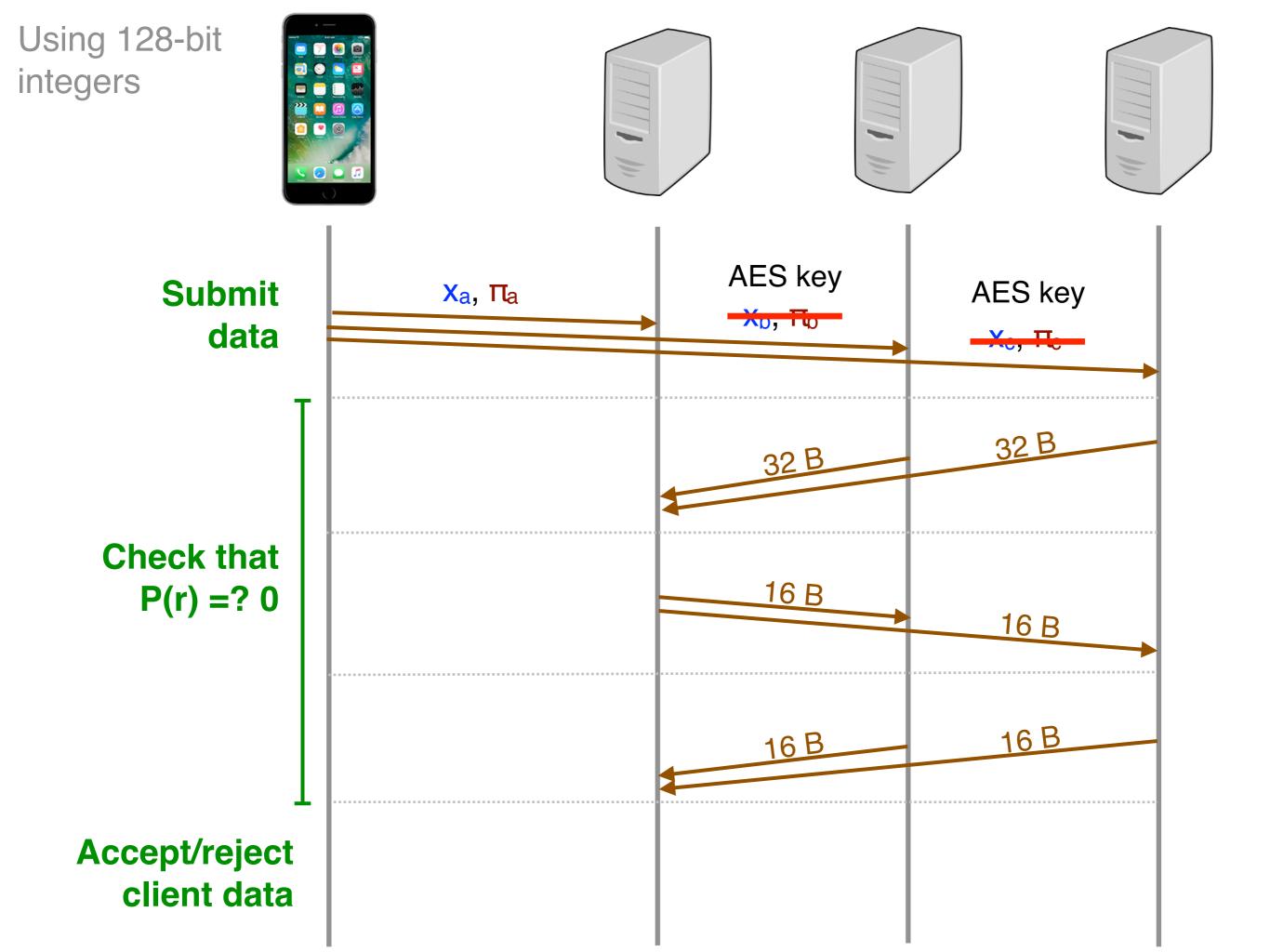


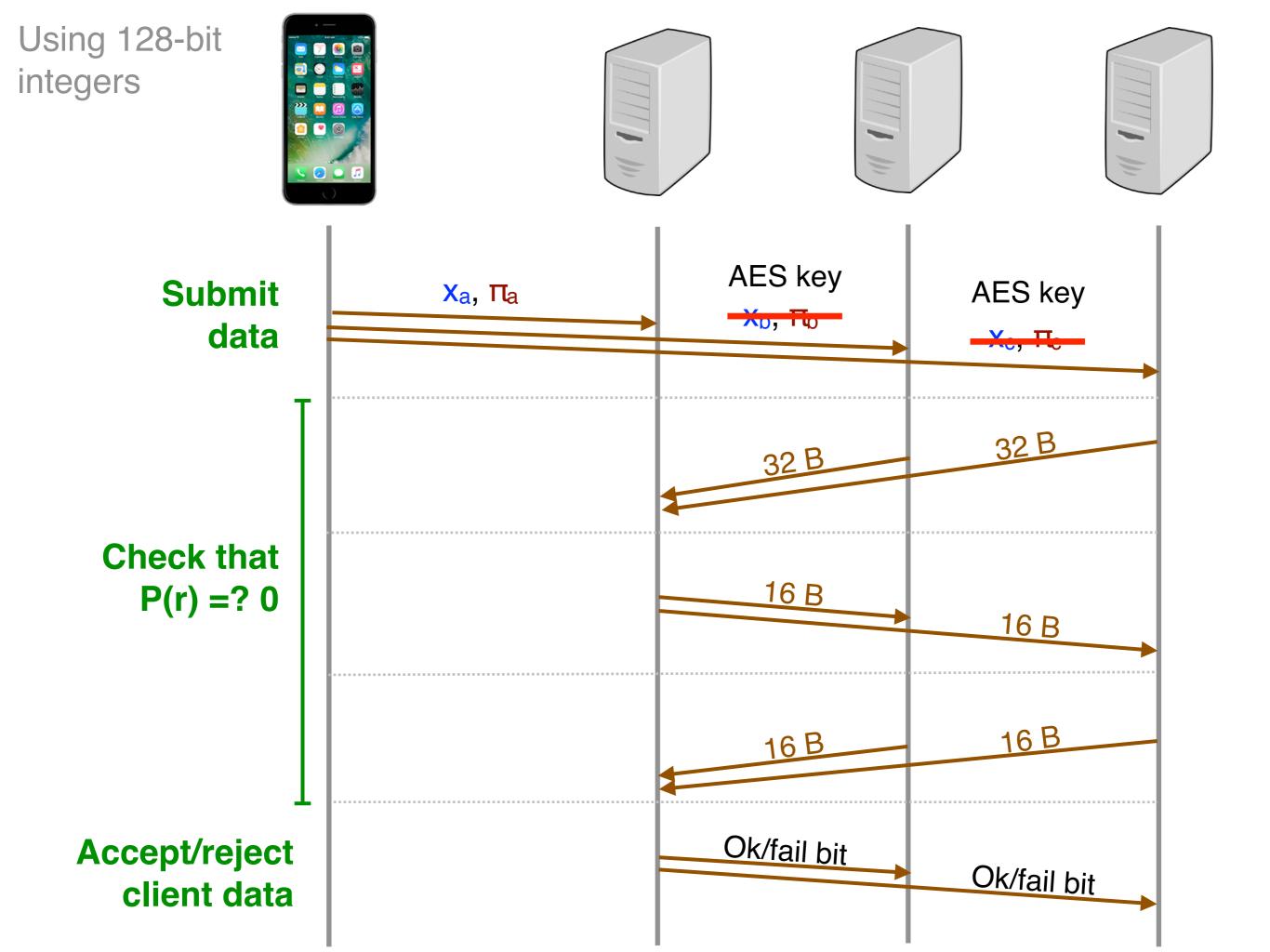


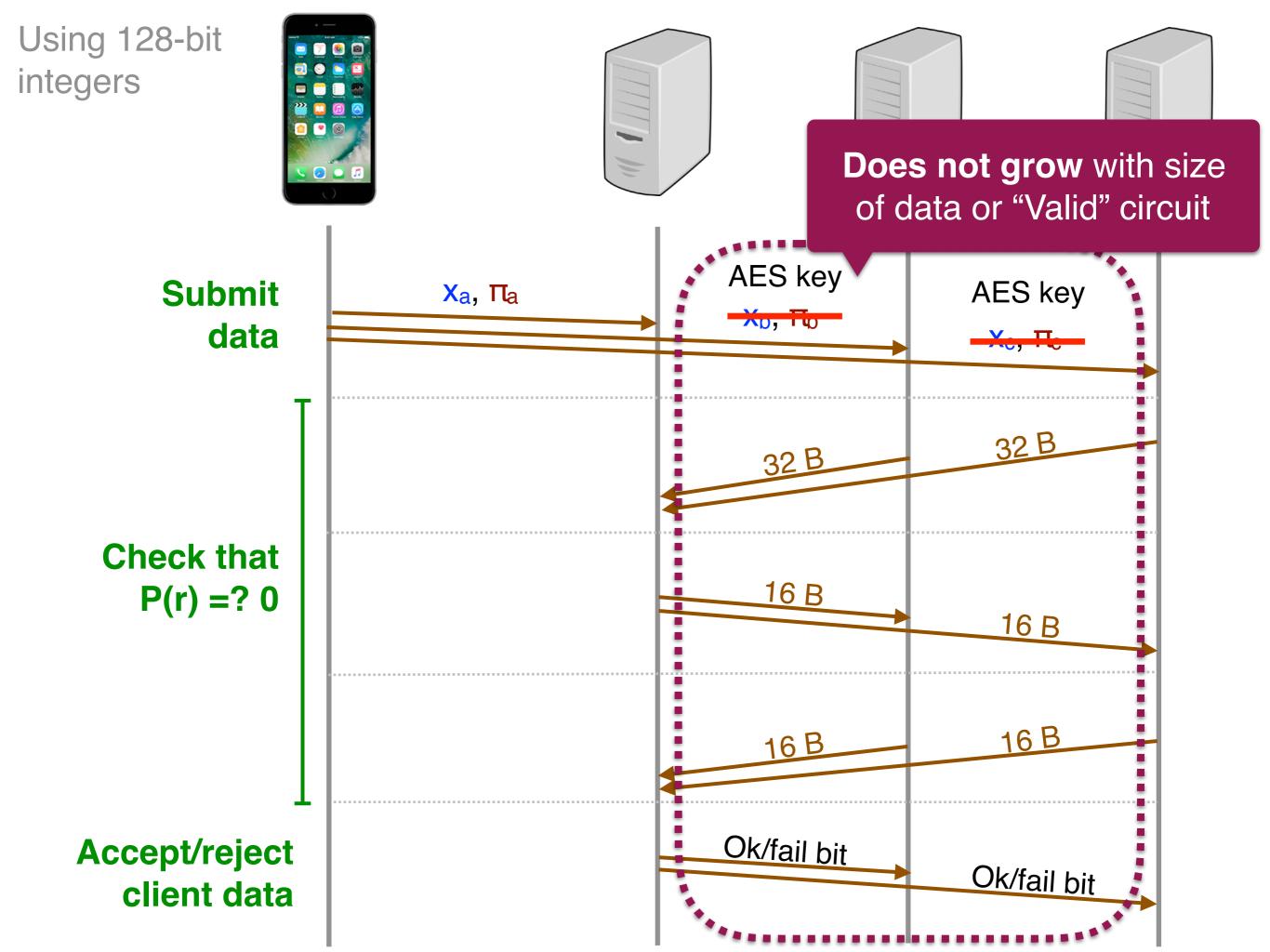


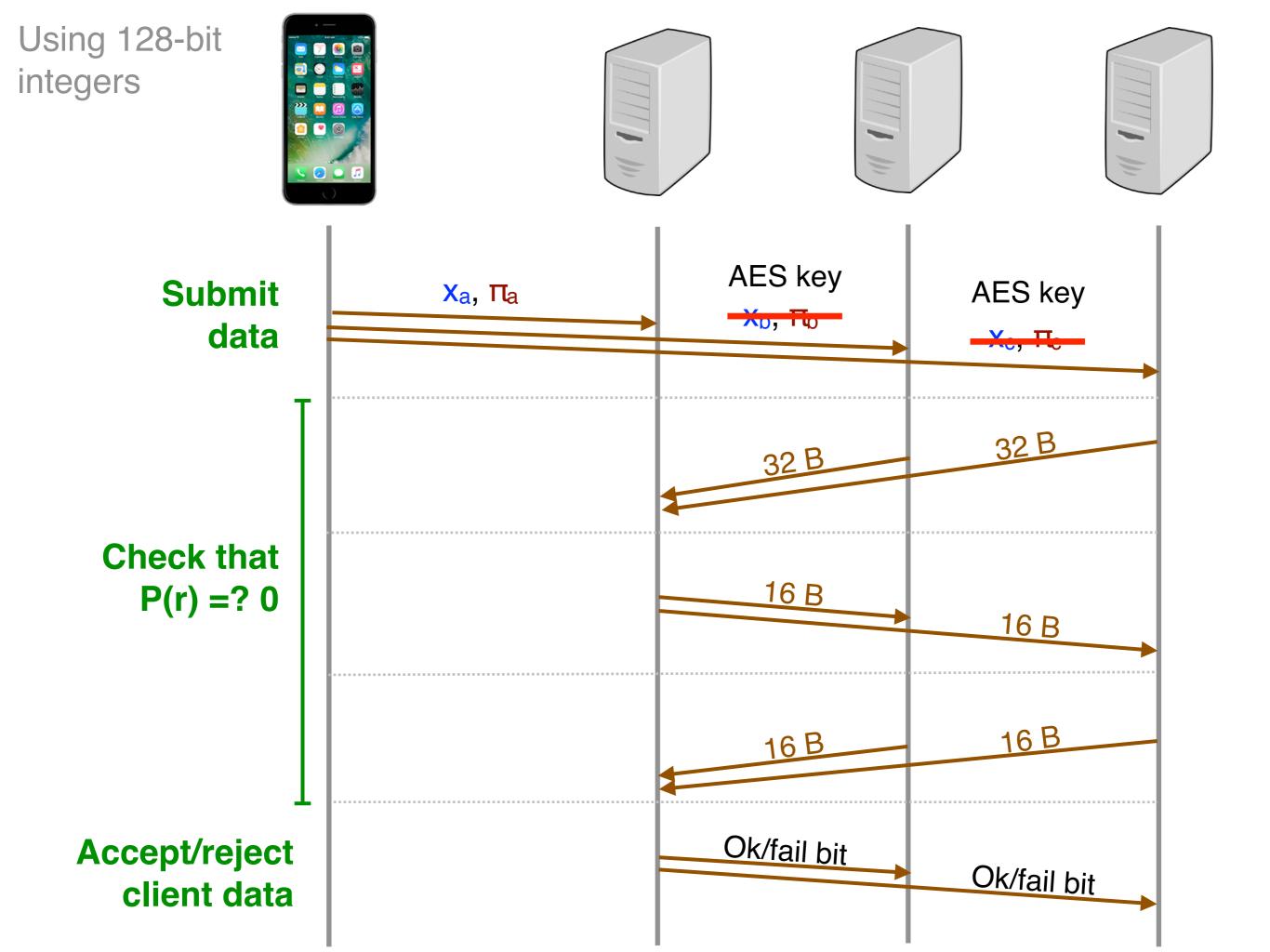












- Each of N clients holds a 4-bit value Xi
- Servers want the AVG and VAR of the xis.

Each client encodes her value  $x = b_3b_2b_1b_0$  as the tuple  $(x, y) = (x, x^2, b_3, b_2, b_1, b_0)$ 

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To test validity of the encoding, check that:

Valid(x, y) = { 
$$(x^2 - y) = 0 - y \text{ is } x^2$$
  
{  $x - \sum_j 2^j b_j = 0 - b$ 's are the bits of x  
{  $b_j \cdot (b_j - 1) = 0 - b$ 's are 0/1 values